Name: Answer Key Panther ID: _____

Worksheet May 18 Trigonometry Summer A 2016

1. Find, without a calculator, the exact value of each of the following. Use the unit circle.

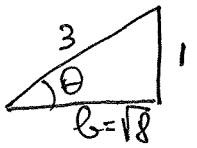
$\sin(\pi/6) = \frac{1}{2}$ $\cos(120°) = \frac{1}{2}$ $\cos(5\pi/4) = -\frac{\sqrt{2}}{2}$ $\tan(5\pi/4) = 1$

$\sec(0) = \frac{1}{\cos(0)} = \frac{1}{1} = 1$ $\csc(150°) = \frac{1}{\sin(150°)}$, $\frac{1}{\frac{1}{2}} = 2$ $\cot(7\pi/2) = 0$, $\frac{\cos(\frac{7\pi}{2})}{\sin(\frac{7\pi}{2})} = \frac{0}{-1} = 0$ $\tan(7\pi/2) =$ undefined, $\frac{\sin(\frac{7\pi}{2})}{\cos(\frac{7\pi}{2})} = \frac{-1}{0}$

For all of the above, you should use the unit circle and a reference angle

2. (a) Given that θ is an angle in the first quadrant and that $\sin(\theta) = \frac{1}{3}$, find, without calculator, the exact value of each of the following

$\cos(\theta) = \frac{\sqrt{8}}{3}$ $\tan(\theta) = \frac{1}{\sqrt{8}}$ $\cot(\theta) = \frac{\sqrt{8}}{1}$ $\sec(\theta) = \frac{3}{\sqrt{8}}$ $\csc(\theta) = \frac{1}{\sin\theta} = 3$



$b^2 + 1^2 = 3^2$

$b^2 = 3^2 - 1^2 = 8 \Rightarrow b = \sqrt{8} = 2\sqrt{2}$ (either one is fine)

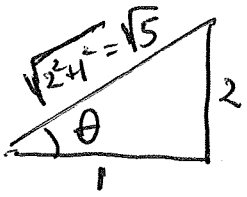
(b) Given that θ is an angle in the second quadrant and that $\sin(\theta) = \frac{1}{3}$, find, without calculator, the exact value of each of the following

$\cos(\theta) = -\frac{\sqrt{8}}{3}$ $\tan(\theta) = -\frac{1}{\sqrt{8}}$ $\cot(\theta) = -\frac{\sqrt{8}}{1}$ $\sec(\theta) = -\frac{3}{\sqrt{8}}$ $\csc(\theta) = 3$

same absolute values as in part (a)
but signs are now determined by θ being in the second quadrant

(c) Given that θ is an angle in the first quadrant and that $\tan(\theta) = 2$, find, without calculator, the exact value of each of the following

$\cos(\theta) = \frac{1}{\sqrt{5}}$ $\sin(\theta) = \frac{2}{\sqrt{5}}$ $\cot(\theta) = \frac{1}{2}$ $\sec(\theta) = \frac{1}{\cos\theta} = \sqrt{5}$

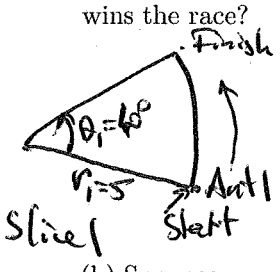


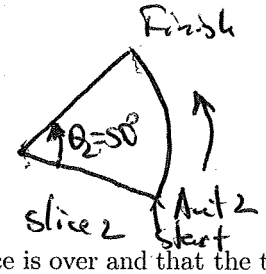
(d) Given that θ is an angle in the third quadrant and that $\tan(\theta) = 2$, find, without calculator, the exact value of each of the following

$\cos(\theta) = -\frac{1}{\sqrt{5}}$ $\sin(\theta) = -\frac{2}{\sqrt{5}}$ $\cot(\theta) = \frac{1}{2}$ $\sec(\theta) = -\sqrt{5}$

Answer for (3a) The race ends in a tie, since the ants cover the same distance and has an angle of $\theta_2 = 50°$

3. (a) Two slices of pizza are sitting on a table. Slice1 is cut from a large pizza with a radius $r_1 = 5$ in and has a central angle $\theta_1 = 40°$. Slice2 is cut from a medium pizza with a radius $r_2 = 4$ in. Two ants, Ant1 and Ant2, are setting up a race (see the picture). Assuming the ants start at the same time and have the same speed, which ant wins the race?





$\theta_1 = 40° = 40 \cdot \frac{\pi}{180} = \frac{2\pi}{9}$ rads.

so $l_{arc_1} = \theta_1 \cdot r_1 = \frac{2\pi}{9} \cdot 5 = \frac{10\pi}{9}$ inches.

$\theta_2 = 50° = 50 \cdot \frac{\pi}{180} = \frac{5\pi}{18}$ rads. so

$l_{arc_2} = \theta_2 \cdot r_2 = \frac{5\pi}{18} \times 4 = \frac{10\pi}{9}$ inches.

(b) Suppose now the race is over and that the two ants are allowed to buy their corresponding slices of pizzas (for winter provisions for their colonies – the ants belong to different colonies :)). Each slice costs \$1. Which ant gets the better deal?

The one getting the slice with bigger area gets the better deal.

Area slice$_1$ $= \frac{1}{2}\theta_1 \cdot r_1^2 = \frac{1}{2} \cdot \frac{2\pi}{9} \cdot 5^2 = \frac{25\pi}{9}$ in^2

Area slice$_2$ $= \frac{1}{2}\theta_2 \cdot r_2^2 = \frac{1}{2} \cdot \frac{5\pi}{18} \cdot 4^2 = \frac{20\pi}{9}$ in^2

Since slice 1 has bigger area, Ant1 gets the better deal.