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**Worksheet June 1**        Trigonometry        Summer A 2016

**1.** (a) Use the Euler formula to obtain formulas for $\cos(\alpha - \beta)$ and $\sin(\alpha - \beta)$.

$$e^{i(\alpha-\beta)} = \frac{e^{i\alpha}}{e^{i\beta}} = \frac{\cos(\alpha)+i\sin(\beta)}{\cos(\beta)+i\sin(\beta)} = (\cos(\alpha)+i\sin(\beta))(\cos(\beta)-i\sin(\beta))$$

$$\parallel$$

$$e^{i\alpha}\cdot e^{-i\beta} = \cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta) + i\sin(\alpha)\cos(\beta) - i\cos(\alpha)\sin(\beta)$$

So, $\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$

and $\sin(\alpha-\beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$.

(b) Use the formulas you found in part (a) to find a formula for $\tan(\alpha - \beta)$ in terms of $\tan\alpha$ and $\tan\beta$.

$$\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \frac{\sin(\alpha)\cos(\beta)-\cos(\alpha)\sin(\beta)}{\cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta)}$$

$$= \frac{\frac{\sin(\alpha)}{\cos(\alpha)} - \frac{\sin(\beta)}{\cos(\beta)}}{1 + \frac{\sin(\alpha)\sin(\beta)}{\cos(\alpha)\cos(\beta)}}$$

$$= \frac{\tan(\alpha)-\tan(\beta)}{1+\tan(\alpha)\tan(\beta)}.$$

(c) Find the exact value of the expression $\cos(55°)\cos(10°) + \sin(55°)\sin(10°)$.

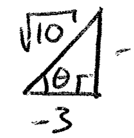
$$\cos(55°)\cos(10°)+\sin(55°)\sin(10°) = \cos(55°-10°) = \cos(45°) = \frac{1}{\sqrt{2}}.$$

**2.** If $\cot\theta = 3$ and $\theta$ lies in the 3rd quadrant, find the exact values of $\sin(2\theta)$, $\cos(2\theta)$, $\tan(2\theta)$.

$$\cot(\theta) = 3 = \frac{\cos(\theta)}{\sin(\theta)}$$

We need to find $\cos\theta$, $\sin\theta$.

We use the $\Delta$ method →




So $\cos\theta = -\frac{3}{\sqrt{10}}$, $\sin\theta = \frac{-1}{\sqrt{10}}$

→ So $\tan\theta = \frac{1}{\cot\theta} = \frac{1}{3}$

$$\tan(2\theta) = \frac{\tan(\theta)+\tan(\theta)}{1-\tan^2(\theta)} = \frac{\left(\frac{2}{3}\right)}{1-\frac{1}{9}} = \frac{2}{3}\cdot\frac{9}{8} = \boxed{\frac{3}{4}}.$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = \left(\frac{-3}{\sqrt{10}}\right)^2 - \left(\frac{-1}{\sqrt{10}}\right)^2 = \frac{9}{10} - \frac{1}{10} = \boxed{\frac{4}{5}}$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta) = 2\left(\frac{-3}{\sqrt{10}}\right)\left(\frac{-1}{\sqrt{10}}\right) = \frac{6}{10} = \boxed{\frac{3}{5}}.$$

for both we choose '+' sign as $22.5^\circ$ is in the $1^{st}$ quadrant

**3.** (a) Use half-angle formulas to find the exact values of $\sin(22.5°)$, $\cos(22.5°)$, $\tan(22.5°)$.

$$\sin(22.5^\circ) = \sin\left(\frac{1}{2}\cdot 45^\circ\right) = \sqrt{\frac{1-\cos(45^\circ)}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}} = \boxed{\frac{\sqrt{2-\sqrt{2}}}{2}}$$

$$\cos(22.5^\circ) = \cos\left(\frac{1}{2}\cdot 45^\circ\right) = \sqrt{\frac{1+\cos(45^\circ)}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \dots = \boxed{\frac{\sqrt{2+\sqrt{2}}}{2}}$$

$$\tan(22.5^\circ) = \frac{\sin(22.5^\circ)}{\cos(22.5^\circ)} = \frac{\frac{\sqrt{2-\sqrt{2}}}{2}}{\frac{\sqrt{2+\sqrt{2}}}{2}} = \frac{\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}$$

(b) Use half-angle formulas to find the exact values of $\sin(\frac{5\pi}{8})$, $\cos(\frac{5\pi}{8})$, $\tan(\frac{5\pi}{8})$.

Signs are chosen so, because $\frac{5\pi}{8}$ is in the 2nd quadrant

$$\sin\left(\frac{5\pi}{8}\right) = \sin\left(\frac{1}{2}\cdot\frac{5\pi}{4}\right) = +\sqrt{\frac{1-\cos(\frac{5\pi}{4})}{2}} = \sqrt{\frac{1-(-\frac{\sqrt{2}}{2})}{2}} = \dots = \frac{\sqrt{2+\sqrt{2}}}{2}$$

$$\cos\left(\frac{5\pi}{8}\right) = \cos\left(\frac{1}{2}\cdot\frac{5\pi}{4}\right) = -\sqrt{\frac{1+\cos(\frac{5\pi}{4})}{2}} = -\sqrt{\frac{1+(-\frac{\sqrt{2}}{2})}{2}} = \dots = -\frac{\sqrt{2-\sqrt{2}}}{2}$$

$$\tan\left(\frac{5\pi}{8}\right) = -\frac{\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}$$

**4.** Find an identity for $\cos(4\theta)$ in terms of $\cos(\theta)$.

$$\cos(4\theta) = \cos(2\cdot(2\theta)) = 2\cos^2(2\theta) - 1$$

$$= 2\left(2\cos^2\theta - 1\right)^2 - 1$$

So $\cos(4\theta) = 2\left(2\cos^2\theta - 1\right)^2 - 1$ or, if we expand,

$$\cos(4\theta) = 8\cos^4\theta - 8\cos^2\theta + 1$$