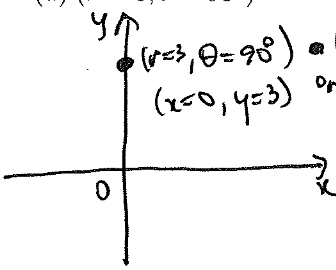
Name: Solution Key Panther ID: ____________

Worksheet June 10 Trigonometry Summer A 2016

1. In each part, you are given the polar coordinates of a point. First plot the point, and then find the rectangular coordinates of each point. Finally, give one different polar coordinates representation of the same point.

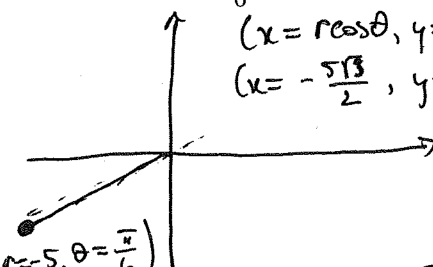
(a) $(r = 3, \theta = 90°)$



$(r=3, \theta=90°)$ $(r=-3, \theta=-90°)$

$(x=0, y=3)$ or $(r=-3, \theta=\frac{3\pi}{2})$

(b) $(r = -5, \theta = \frac{\pi}{6})$



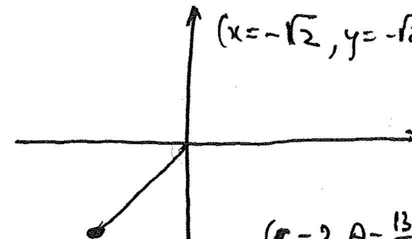
$(x = r\cos\theta,\ y = r\sin\theta)$

$(x = -\frac{5\sqrt{3}}{2},\ y = -\frac{5}{2})$

$(r=-5, \theta=\frac{\pi}{6})$

or $(r=5, \theta=\frac{7\pi}{6})$ or $(r=5, \theta=-\frac{5\pi}{6})$

(c) $(r = 2, \theta = \frac{5\pi}{4})$



$(x=-\sqrt{2}, y=-\sqrt{2})$

$(r=2, \theta=\frac{5\pi}{4})$ or $(r=2, \theta=\frac{13\pi}{4})$

or $(r=-2, \theta=-\frac{3\pi}{4})$

2. Convert each rectangular equation to a polar equation that expresses r in terms of θ.

(a) $x^2+y^2 = 25$

$r^2=25$

$\boxed{r=5}$

(b) $(x+3)^2+y^2 = 9 \iff$

$x^2+6x+9+y^2=9 \iff$

$\iff x^2+y^2+6x=0 \iff$

$\iff r^2+6r\cos\theta=0 \iff$

$\iff \boxed{r=-6\cos\theta}$

(c) $x^2 = 3y$

$r^2\cos^2\theta = 3r\sin\theta \quad |\div r$

$r\cos^2\theta = 3\sin\theta \quad \div \cos^2\theta$

$r = 3\frac{\sin\theta}{\cos^2\theta}$ or

$\boxed{r = 3\tan\theta\sec\theta}$

3. Convert each polar equation to a rectangular coordinate equation.

(a) $r\sin\theta = -3$

$\boxed{y=-3}$

(b) $r = 2\cos\theta \quad |\cdot r$

$r^2 = 2r\cos\theta$

$\boxed{x^2+y^2 = 2x}$

or, after completing square,

$(x-1)^2+y^2=1$

(c) $r^2\sin(2\theta) = 6$

$r^2 2\sin\theta\cos\theta = 6 \quad |\div 2$

$(r\cos\theta)\cdot(r\sin\theta) = 3$

$\boxed{x\cdot y = 3}$

4. Convert to rectangular coordinates to show that the graph of $r = a\cos\theta$ is a circle with center at $(a/2, 0)$ and radius $a/2$.

$r = a\cos\theta \iff r^2 = ar\cos\theta \iff x^2+y^2 = ax \iff$

$\iff x^2-ax+y^2=0 \iff x^2-2\cdot\frac{a}{2}x+\left(\frac{a}{2}\right)^2-\left(\frac{a}{2}\right)^2+y^2=0$

$\iff \left(x-\frac{a}{2}\right)^2+y^2 = \left(\frac{a}{2}\right)^2$

so the curve is a circle with center at $\left(\frac{a}{2}, 0\right)$ and radius $\frac{a}{2}$.