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Worksheet - June 20

Trigonometry

Summer A 2016

1. For each given complex number, plot the number and then find a polar form.

(a)  $z = 3+3i$

(b)  $z = 5i$

(c)  $z = 2-i\sqrt{3}$

(d)  $z = -4$

2. Given a complex number  $z = a + ib$ , the number  $\bar{z} = a - ib$  is called the complex conjugate of  $z$ .

(a) Show that  $z \cdot \bar{z} = |z|^2$  where  $|z| = r = \sqrt{a^2 + b^2}$  is the modulus of the complex number  $z$ .

(b) Use (a) to show that the quotient of two complex numbers  $\frac{z_1}{z_2}$  can be computed by

$$\frac{z_1}{z_2} = \frac{z_1 \cdot \bar{z}_2}{z_2 \cdot \bar{z}_2} = \frac{z_1 \cdot \bar{z}_2}{|z_2|^2}$$

(c) Use (b) to find the quotient  $\frac{z_1}{z_2}$  if  $z_1 = 1 + i$ ,  $z_2 = 2 - i\sqrt{3}$ . Repeat for  $z_1 = -3 + 4i$ ,  $z_2 = -3 - 4i$ .

(d) If  $z = re^{i\theta}$ , what is the polar form of the conjugate  $\bar{z}$ ?

(e) Use (d) to find the quotient  $\frac{z_1}{z_2}$  if  $z_1 = 3\left(\cos \frac{\pi}{5} + i \sin \frac{\pi}{5}\right)$ ,  $z_2 = 4\left(\cos \frac{\pi}{10} + i \sin \frac{\pi}{10}\right)$ . Answer in polar form.

3. Use polar form and DeMoivre formula to find  $(1 + i)^{10}$ .

4. (a) Find all complex solutions of  $z^5 = 1$  and plot the solutions on a rectangular coordinate system.

(b) Find all complex solutions of  $z^4 = 25i$  and plot the solutions on a rectangular coordinate system.

(c) Find the complex third roots of  $1 + i$  and plot them.