Optimum allocation of weights to assets in a portfolio: the case of nominal annualisation versus effective annualisation of returns

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ABSTRACT

Since the introduction of modern portfolio theory by Roy (1952), Markowitz (1952) and Sharpe (1964) about half a century ago, the allocation of investment weights among various assets in a portfolio is one of the most important areas of research in finance. However, we are not aware of any study that has compared the allocation problem under nominally annualised versus effectively annualised returns. In this paper, we empirically examine the effect of effective annualisation on the variance and skewness of the rates of return probability distribution and the allocation of weights to assets in the portfolio. Our empirical findings conclude that the method of annualisation drastically affects the variance and skewness as well as the allocation of weights to the assets in the portfolio.

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1. Introduction

In two recent studies, namely, Chunhachinda, Dandapani, Hamid & Prakash (1997) and Prakash, Chang & Pactwa (2003), the authors empirically examine the portfolio allocation in the presence of investor's positive skewness preferences. In both studies, they use several world capital markets as their empirical sample. In general, they conclude that if the investors allocate their wealth to the assets in the portfolio on the basis of mean-variance-positive-skewness preferences in the rates of return probability distributions, the allocation of funds to various assets in the portfolio changes significantly from that of just under mean-variance portfolio allocations. The methodology used in both the above cited studies is developed by Lai (1991) and Meric and Meric (1989).

One of the differences between Chunhachinda et al. (1997) and Prakash et al. (2003) is that the former study does not consider the allocation to the risk free asset in their portfolio construction whereas the latter study allows the allocation of funds in the risk free assets. Both studies construct the allocation with weekly as well as monthly data. However, there is a big difference between these two studies. Chunhachinda et al. (1997) consider the portfolio allocation using raw weekly and monthly returns whereas Prakash et al. (2003) perform the allocation using effective annualised weekly returns as well as effective annualised monthly returns[1]. On the surface, this may not seem to be a point of concern. However, the effective annualisation of weekly and monthly returns completely changes the *direction* of variance and skewness in the probability distribution of the rates of return. For example, Prakash, de Boyrie, Hamid and Smyser (1997) theoretically show that the choice of investment horizon (e.g. daily versus weekly versus monthly, etc.) affects the magnitude of the variance and skewness. In brief, they show that the variance of rates of return increases with the investment horizon. Thus, the variance of daily rates of return is smaller than the variance of weekly rates of return; the variance of weekly rates of return is smaller than the variance of monthly rates of return; etc. Similarly, they also prove that the magnitudes of skewness measures follow the same relationship. That is, skewness of daily rates of return is smaller than that of weekly rates of return, which is smaller than the skewness of monthly rates of return, etc.

In this paper, we theoretically and empirically demonstrate that if, however, the rates of returns are annualised using effective annualisation (not nominal annualisation), the relationship developed in Prakash, deBoyrie, Hamid and Smyser (1997) is completely reversed. That is, for effective annual rates of return, we show that the variance (as well as skewness measure) of an annualised daily return is larger than that of an annualised weekly return; and the variance (as well as skewness measure) of an annualised weekly return is larger than that of an annualised monthly return; etc. Therefore, it is obvious that whatever portfolio allocation achieved using nominal rates of return (as done by Chunhachinda et al. and Prakash et al.) may be quite different. In this paper, we investigate the portfolio allocation using the effective annualisation of the rates of return and find that the allocation using the nominal rates of return is quite different than what we obtain using the annualised rate of return.

In this paper, we refrain from providing extensive literature review as well as details concerning the methodology used in portfolio allocation. The literature review can be found in Chunhachinda et al. (1997) and Prakash et al. (2003).

Similarly, we replicate the procedure used by Chunhachinda et al. (1997) in their section 2, page 145. The paper is organized as follows.

The theoretical development of the effect of annualisation is presented in section 2. In section 3 we describe the data collected for our empirical investigations. The empirical results of annualisation effect are discussed in section 4. The empirical findings using 37 global equity indexes are also presented in Section 5. Some concluding remarks are included in Section 6.

2. The effect of annualisation on variance and skewness of the rates of return probability distribution[2]

Assume that $\tilde{P}_j$ and P_{j-1} are respectively the expected price (a random variable), and the known price (non-random number) to prevail in time period j and j-1, then the one-period random rate of return during the interval (j-1) to j will be [3]

$$\tilde{r}_j = \frac{\tilde{P}_j - P_{j-1}}{P_{j-1}} \tag{1}$$

$$\text{or } \tilde{P}_j - (1+\tilde{r}_j)P_{j-1} = 0 \tag{2}$$

Putting j = 1,2,……,T and solving recursively we get

$$\tilde{P}_T = P_0(1+\tilde{r}_1)(1+\tilde{r}_2)......(1+\tilde{r}_T) \tag{3}$$

or $$\ln\frac{\tilde{P}_T}{P_0} = \sum_{j=1}^{T}\ln(1+\tilde{r}_j)$$

and $$\ln\tilde{R}_0^T = \sum_{j=1}^{T}\ln\tilde{R}_j \tag{4}$$

where $\tilde{R}_0^T$ denotes the holding period (from 0 to T) wealth ratio.

The interest is in the probability distribution of $\ln \tilde{R}_0^T$. Prakash, de Boyrie, Hamid and Smyser (1997) show that expression (4) given by $Y = \ln \tilde{R}_0^T = \sum_{j=1}^{T} \ln \tilde{R}_j$ is asymptotically normally distributed, therefore, $x = \prod_{j=1}^{T} \tilde{R}_j$ will be lognormal with parameters[4] $\theta = \sum_{j=1}^{T} \mu_j$ and $\xi^2 = \sum_{j-1}^{T} \sigma_j^2$ where μ_j and σ_j^2 are the mean and variance of R_j (j = 1,2,…,T). That is, the probability density function of $x = \prod_{j=1}^{T} \tilde{R}_j$ is given by

$$f(x) = \frac{1}{x\xi\sqrt{2\pi}} \exp\left\{-\frac{1}{2\xi^2}(\ln x - \theta)^2\right\}, \quad 0 < x < \infty \qquad (5)$$

The mean, variance and skewness of the probability distribution of x are given by

Mean: $M = \exp\left(\theta + \frac{\xi^2}{2}\right) - 1$

Variance: $V = \exp(2\theta + \xi^2)\left[\exp\left(\xi^2\right) - 1\right]$ (6)

Skewness: $SK = \left[\exp\left(\xi^2\right) - 1\right]^{3/2} + 3\left[\exp\left(\xi^2\right) - 1\right]^{1/2}$

Now suppose that we have n periods within a year and wish to study the skewness of the corresponding *effectively annualised returns*. The gross return for a given period can be expressed as $x = \prod_{j=1}^{T} \tilde{R}_j$ where the sub-periods j = 1…T are taking place "inside" that given period. We must then determine the probability density function of $x^n = \left[\prod_{j=1}^{T} \tilde{R}_j\right]^n$ or the probability density function of $\left[\tilde{R}_0^T\right]^n$ assuming that the returns are independent and identically distributed. For example, if x is a monthly return, that is, T = 30, we then have n = 12 and can think of the sub-periods j as days within that month for instance. Let us define $y = g(x) = x^n$. We can use the well known change-of-variable formula to determine the density function of y, denoted by $f_y(y)$, as

$$f_y(y) = \left|\frac{dx}{dy}\right| \cdot f_x\left[g^{-1}(y)\right] \tag{7}$$

Since $x = y^{\frac{1}{n}}$, then $\left|\frac{dy}{dx}\right| = \frac{1}{n} y^{\frac{1}{n}-1}$. Substituting these into equation (11) and using equation (4), we obtain

$$\begin{aligned} f_y(y) &= \frac{1}{n} y^{\frac{1}{n}-1} \cdot \frac{1}{y^{\frac{1}{n}} \xi \sqrt{2\pi}} \exp\left\{ -\frac{1}{2\xi^2} \left[\ln\left(y^{\frac{1}{n}} \right) - \theta \right]^2 \right\} \\ &= \frac{1}{y(n\xi)\sqrt{2\pi}} \exp\left\{ -\frac{1}{2(n\xi)^2} \left[\ln(y) - n\theta \right]^2 \right\} \end{aligned} \tag{8}$$

Note that equation (8) is also the probability density function of a lognormal distribution with parameters $n\xi$ and $n\theta$.

That is,

$$y = \; x^n = \left[\prod_{j=1}^{T} \tilde{R}_j \right]^n \backsim \text{lognormal } (n\theta, n^2\xi^2). \tag{9}$$

If we assume that $\tilde{R}_j$ pertains to one day and since $\ln \tilde{R}_j$'s are independently and identically distributed, then, $\theta = T\mu$ and $\xi^2 = T\sigma^2$, or

$$\tilde{R}_0^T \sim \text{Lognormal}\left(\theta = T\mu,\ \xi^2 = T\sigma^2 \right)$$

Thus, we can rewrite equation (9) as

$$x^n = \left[\prod_{j=1}^{T} \tilde{R}_j \right]^n \backsim \text{Lognormal } (nT\theta, n^2T\xi^2). \tag{10}$$

Following the procedure given in Prakash, Hamid, deBoyrie and Smyser (1997), we can obtain the mean, variance and skewness of the probability distribution of x^n as:

$$\text{Mean: } M' = \exp\left(nT\mu + \frac{n^2T\sigma^2}{2}\right) - 1$$

$$\text{Variance: } V' = \exp(2nT\mu + n^2T\sigma^2)\left[\exp\left(n^2T\sigma^2\right) - 1\right] \tag{11}$$

$$\text{Skewness: } SK' = \left[\exp\left(n^2T\sigma^2\right) - 1\right]^{3/2} + 3\left[\exp\left(n^2T\sigma^2\right) - 1\right]^{1/2}$$

Recall that we define *T* as the investment horizon and *n* as the multiple required to annualise the return (for example, if $T = 30$ then $n = 12$), we have $n \cdot T = 360$. Substitute this relationship into equations (11), we obtain

$$\text{Mean: } M' = \exp\left(360\mu + \frac{360n\sigma^2}{2}\right) - 1$$

$$\text{Variance: } V' = \exp(720\mu + 360n\sigma^2)\left[\exp\left(360n\sigma^2\right) - 1\right] \tag{12}$$

$$\text{Skewness: } SK' = \left[\exp\left(360n\sigma^2\right) - 1\right]^{3/2} + 3\left[\exp\left(360n\sigma^2\right) - 1\right]^{1/2}$$

It is evident from equations (12) that with effective annualisation, both the variance and skewness of the probability distribution are strictly increasing functions of *n*. Hence, after effective annualisation, the daily returns have higher variance and skewness than the weekly returns; weekly returns are more variant and skewed than the monthly ones; and so on. We conclude that for effectively annualised rate of return

$$V\left[(\tilde{R}_0^1)^{360}\right] > V\left[(\tilde{R}_0^7)^{52}\right] > V\left[(\tilde{R}_0^{30})^{12}\right] > V\left[(\tilde{R}_0^{90})^4\right] > V\left[(\tilde{R}_0^{180})^2\right] > V(\tilde{R}_0^{360}) \tag{13}$$

where the difference between the superscript and the subscript denotes the investment horizon. Similarly the same inequality as obtained for variance will be maintained for skewness expressed as:

$$SK\left[(\tilde{R}_0^1)^{360}\right] > SK\left[(\tilde{R}_0^7)^{52}\right] > SK\left[(\tilde{R}_0^{30})^{12}\right] > SK\left[(\tilde{R}_0^{90})^4\right] > SK\left[(\tilde{R}_0^{180})^2\right] > SK(\tilde{R}_0^{360}) \tag{14}$$

Thiese results are just the opposite of what Prakash et. el. Found in their study. Thatis for raw as well as nominal rates of return the behaviour of skewness and variance are as follows:

For variance:

$$V(\tilde{R}_0^1) \leq V(\tilde{R}_0^5) \leq V(\tilde{R}_0^{10}) \leq V(\tilde{R}_0^{20}) \leq V(\tilde{R}_0^{40})$$ (

For skewness:

$$SK(\tilde{R}_0^1) \leq SK(\tilde{R}_0^5) \leq SK(\tilde{R}_0^{10}) \leq SK(\tilde{R}_0^{20}) \leq SK(\tilde{R}_0^{40})$$ “

(10)

The data

We use world markets index data from July 1993 to May 2005. In contrast with Prakash et al. (2003) who used only 17 countries, we include 37 countries spanning over the five continents. We also use monthly as well as weekly returns in order to investigate the effect of annualisation in portfolio allocation. The price series for each country index were obtained from Datastream and subsequently converted to return series. The countries considered are

Developed countries:

Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Norway, Spain, Sweden, Switzerland, UK, Australia, Canada, New Zealand, US, Japan, Singapore, and Hong Kong.

Developing countries:

China, Indonesia, Korea, Malaysia, Philippines, Taiwan, Thailand, Argentina, Brazil, Chile, Mexico, Venezuela, Portugal, Turkey, and Poland.

3. The empirical verification of the effect of annualisation on variance and skewness

According to expressions (13) and (14), the variance and skewness for effective annualised returns should increase as the investment horizon decreases.

[Insert table 1A here]

For the EAR returns in Table 1A, the variance and skewness measures are exactly as predicted by the theoretical construct. That is, all the computed values of weekly variances and skewnesses are higher than their monthly counterparts. The computed values of variances and skewnesses for EAR returns are completely in conformity with the theoretically predicted directions. That is, the weekly measures in both cases are higher than their monthly counterparts for each country. In addition, the results of the W-test show that the null hypothesis of normality is rejected for each of the capital markets. In fact, for weekly EAR results, the probability of significance is near zero for each and every one of the 37 capital markets. In case of monthly returns, the trend is also maintained, but the results are somewhat weaker in comparison to weekly EAR returns.

[Insert table 1B here]

For the nominally annualised returns, the summary statistics are presented in Table 1B. Under nominal annualisation, as in case of effective annualisation, the variances computed from annualised weekly raw returns are generally higher than the variances obtained from annualised monthly raw returns. However, we can not draw the same conclusion for skewness measures. Table 1B shows that roughly half of the countries in our sample have their weekly skewness measures outweigh their monthly counterparts. In terms of W-statistics results, we also find a very different picture from the case of EAR returns. That is, for nominally annualised weekly returns, there is only one country in our sample, namely, Austria, where we can not reject the normality null hypothesis at the five percent significance level. However for the nominally annualised monthly raw returns, we can not reject the normality null hypothesis. for 15 out of 37 countries in our sample

5. Empirical results – multi-objective goal programming portfolio allocations[5]

Using the procedure described above, we obtain the portfolio allocation for the twenty-two developed and fifteen developing countries' market indices. Thus, we consider the portfolio allocation among thirty-seven market portfolios.

As mentioned earlier, we refrain from providing the literature review as this has been covered exhaustively in the papers by Chunhachinda et al. (1997) and Prakash et al. (2003). Furthermore, since we are using exactly the same procedure as Chunhachinda et al. (1997), we refrain from repeating the same as well.

5.1 Portfolio allocation among the markets in developed countries

In Table 2A, we provide the optimal portfolio allocation among the developed markets for annualised weekly returns, using both effective annualisation and nominal annualisation. For illustration purpose, we only report the optimal portfolio allocations for two sets of parameter values. They are: a = 1 and b = 0 representing mean-variance portfolio allocation; and a = 1, b = 1, the mean-variance-skewness portfolio allocation[6].

[Insert tables 2A here]

The differences in allocation of the weights in different markets are pronounced if we take the effective and nominal annualisation on the weekly returns. For example, in the developed markets, under mean-variance preference (a = 1, b = 0) and effectively annualised weekly returns, there are nine out of 22 markets where the weights are allocated with Austria (30.79%) receiving the highest allocation followed by Denmark (28.55%). However, in case of nominally annualised weekly returns, the allocation spreads over eight developed markets, with the highest weight in Demark (28.00%) followed by Ireland (21.02%). Thus, it is obvious that the return annualisation method does have an impact on the allocation weights among different markets.

For mean-variance-skewness preference allocation (*a* = 1, *b* = 1), we find a startling result. For effectively annualised weekly returns, the allocation of the weights happens only in Singapore (100%). For nominally annualised weekly returns, the allocation of weights is spread over five developed markets, with almost half of the allocation goes to Italy (47.57%), followed by about a quarter of the funds goes to Switzerland (27.10%). That compared to mean-variance preference (*a* = 1, *b* = 0) allocation the number of countries where the weights are allocated in many markets, under mean variance-skewness preference (*a* = 1, *b* = 1), the number of countries with no weights has dramatically increased. In addition, the effective annualisation results in allocating all the funds to one single market while the nominal annualisation spreads the resources over five markets.

Table 2B illustrates the optimal portfolio weights among developed markets for both effectively and nominally annualised monthly returns.

[Insert tables 2B here]

In the developed markets, under mean-variance preference (*a* = 1, *b* = 0) and effectively annualised monthly returns, the polynomial goal programming technique selects 15 out of 22 developed markets in the optimal portfolio; with US leading the allocation at 26.78%, followed 20.50% allocated to Denmark. In case of nominally annualised monthly returns, the optimal portfolio includes only eight countries, with Denmark (33.97%) getting the highest allocation and US (27.76%) the second highest. Under mean variance-skewness preference (*a* = 1, *b* = 1), we find the same startling results as in the previous case That is, the preference for skewness reduces the optimal allocation to only one market, namely Greece (100%). However, under effective annualisation the funds are allocated to three markets (Hong Kong, 56.15%, Switzerland, 40.97% and Austria, 2.87%). Thus, we again conclude that the annualisation method as well as the investment horizon results in very different portfolio allocation with and without the presence of preference over skewness.

5.2 Portfolio allocation among the markets in developing countries

The weight allocations of assets in the developing markets are presented in Tables 3A (annualised weekly returns) and 3B (annualised monthly returns).

[Insert table 3A here]

Under the mean-variance preference (*a* = 1, *b* = 0), for the effectively annualised weekly returns, only one country, namely, Indonesia, failed to be the part of the optimal portfolio as no weight is assigned to it. Other than this, the optimal weights are fairly evenly allocated among other developing countries, with China getting the highest weight at 12.89% and Thailand (2.44%) the lowest. For the nominally annualised weekly returns, only eight out of 15 countries are selected, and the weights are unevenly assigned. The optimal portfolio is dominated by three countries, with Portugal getting 48.07%, followed by 22.01% in China and 16.07% in Turkey. Nevertheless, the fact remains that when we use the effectively annualised weekly returns, more countries are included in the mean-variance preference framework. In case of mean-variance-skewness preference (*a* = 1, *b* = 1), the spread of weights in the effectively annualised weekly return is over five out of 15 countries, with the highest allocation going to Indonesia at 58.45%, and lowest to

Brazil at 0.51%. On the other hand, the optimal portfolio contains seven countries when the weekly returns are nominally annualised. That portfolio is dominated by China (80.54%), with Portugal making a distant second at 7.86%.

[Insert table 3B here]

In Table 3B, under the mean-variance preference ($a = 1$, $b = 0$), the allocation with respect to the number of countries does not change very much when we use the effectively annualised monthly returns rather than the weekly returns to construct the portfolio. Here, 13 out of 15 countries become part of the optimal portfolio; only Philippines and Thailand are left out. Portugal (37.31%) has the highest weight followed by Mexico at 15.00%. The lowest weight, except the two which have no weights assigned (Philippines and Thailand), is assigned to Malaysia at 0.50%. In both effectively annualised weekly returns as well as effectively annualised monthly returns, the number of countries selected in the portfolio is quite large compared to the number of countries selected in the developed markets. However, with the nominally annualised monthly returns, only six out of 15 countries are included in the optimal portfolio. Again, the highest weights goes to Portugal (45.23%) and the second highest is China at 22.01%.

In case of mean-variance-skewness preference ($a = 1$, $b = 1$), the portfolio weights in case of the effectively annualised monthly covers three out of 15 countries, with almost all the resources concentrated in Philippines (97.78%). In case of nominally annualised monthly returns, the optimal portfolio has four countries and is dominated by China (90.28%). Under the mean-variance-skewness preference, the number of countries allocated a non-zero weight is greatly reduced compared to the base case of mean-variance preference. Once again, we confirm that the annualisation effect distorts the portfolio allocation in developing markets.

5.3 Portfolio allocation among all markets: developed and developing countries

The portfolio allocation for assets in all 37 developed and developing markets is presented in Table 4A (annualised weekly returns) and 4B (annualised monthly returns)...

[Insert table 4A here]

For mean-variance preference (a = 1, b = 0), the allocation for effectively annualised weekly returns covers 21 countries with the highest weights on Austria (29.67%), Denmark (22.38%) and Italy (12.63%). In case of nominally annualised weekly returns, the optimal portfolio is spread over 12 countries with the same three countries, namely, Austria (25.18%) Denmark (23.52%) and Italy (16.78%), leading the pack. In this case, the nominal annualisation seems to reduce the numbers of markets in the optimal portfolio.

[Insert table 4B here]

We observe a similar phenomenon in Table 4B: in case of effectively annualised monthly returns, the allocation of assets is spread among 19 countries with the highest allocation on Austria (25.83%), China (18.89%) and Ireland (13.85%). Note that the change from annualised weekly returns to annualised monthly returns slightly reduces the number of markets selected and changes the composition of the optimal portfolio. In case of nominally annualised monthly returns, the number of countries in the optimal portfolio reduces to 12. This time US has the highest weight at 21.02% and Denmark is second at 20.61%.

For the mean-variance-skewness framework (a = 1, b = 1), the optimal portfolio selections show a very different picture. In the case of effectively annualised weekly returns, only two countries are selected (China and Netherlands), with almost all the funds, 99.99% of them, allocated to Netherlands; whereas in case of nominally annualised weekly returns, three different countries (Ireland, 56.30%, Spain, 31.03% and Norway, 12.67%) are chosen in the portfolio. For effectively annualised monthly returns the optimal mean-variance-skewness portfolio includes four countries, namely Greece, Japan, China and Philippines, with the highest concentration of assets in Philippines (92.41%). On the other hand, if nominal annualisation is used, the optimal portfolio consists of two distinct countries, Hong Kong (60.53%) and Switzerland (39.47%). When we take all markets into consideration, the preference towards skewness seems to reduce the number of markets in the optimal portfolio and allocate most of the assets in one single market, namely, Netherlands, 99.99% in case of effectively annualised weekly returns; and Philippines, 92.41%, in case of effectively annualised monthly returns.

6. Concluding remarks

In this paper, we provide the theoretically predicted behavior of variance and skewness for effective annualised rates of return. In the case of EAR, the theory predicts that the variance as well as skewness will *decrease* as the investment horizon increases. In other words, variance as well as skewness for daily rates of return, when effectively annualised, will be greater than the variance and skewness of say, weekly effectively annualised rates of return. Our empirical results support these theoretical findings.

Like previous studies by Chunhachinda et al. and Prakash et al., we use polynomial goal programming for the portfolio allocation for developing as well developed markets. We find that the annualisation of returns in portfolio selection changes not only the allocation weights, but the number of markets in the portfolio as well. These differences become more pronounced when we move from weekly to monthly to annual returns. In essence, we can say that the impact of annualisation changes not only the assets (capital markets) but the allocation as well. This phenomenon is observed for developing as well as developed countries.

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TABLE 1A Summary statistics - EAR

	mean return		variance		skewness		kurtosis		W-statistic		
	Weekly	**Monthly**	**Weekly**	**Monthly**	**Weekly**	**Monthly**	**Weekly**	**Monthly**	**Weekly**	**Monthly**	**Weel**
AUSTRIA	0.39779	0.25782	1.21930	0.46682	1.95080	1.10190	7.92120	4.05720	10.38600	5.03980	0.00(
BELGIUM	0.57148	0.24664	8.78010	0.36384	13.81900	0.85845	233.46000	3.96840	13.85100	3.64400	0.00(
DENMARK	0.49628	0.30972	1.74980	0.49773	2.54310	1.32930	12.57400	6.18460	11.05500	4.90560	0.00(
FINLAND	4.80830	1.23920	2593.80000	15.78800	18.75300	5.53290	372.57000	39.52200	14.42400	9.35660	0.00(
FRANCE	0.60957	0.28935	6.48160	0.57808	9.21100	1.21860	110.40000	4.35430	13.45000	5.38810	0.00(
GERMANY	0.69718	0.29516	10.65800	0.74242	10.95400	2.59560	148.66000	15.10600	13.70600	6.87070	0.00(
GREECE	2.83460	0.83984	251.64000	7.89030	9.60000	5.59260	105.11000	42.00500	14.07900	9.20010	0.00(
IRELAND	0.64091	0.33513	10.28700	0.60800	17.33000	1.64440	370.51000	7.44060	13.89500	5.76420	0.00(
ITALY	1.86210	0.46948	486.96000	1.93950	20.20800	2.89370	435.76000	14.47700	14.43400	7.85010	0.00(
NETHERLANDS	0.68419	0.24492	31.41000	0.36904	21.21800	0.65265	488.48000	3.48840	14.29100	2.68490	0.00(
NORWAY	0.91178	0.39896	32.55000	0.86344	15.96800	1.24710	290.18000	4.51810	14.14300	5.47330	0.00(
SPAIN	0.77883	0.40394	17.58800	1.26320	17.06000	3.77380	352.69000	26.64000	13.99200	7.87180	0.00(
SWEDEN	5.05000	0.57565	9493.50000	2.13380	24.67700	3.00030	611.55000	16.10000	14.51600	7.65850	0.00(
SWITZERLAND	0.61353	0.28020	11.10700	0.51049	13.78900	1.94640	221.57000	11.05900	13.92700	5.94080	0.00(
UK	0.40092	0.18497	3.82800	0.26161	14.63300	0.50711	286.63000	2.58520	13.60100	2.88570	0.00(
AUSTRALIA	0.48248	0.29689	2.54440	0.60239	5.92940	1.29130	63.72500	4.81190	12.40400	5.44510	0.00(
CANADA	0.54102	0.30703	3.79310	0.50360	7.56390	1.01190	85.86900	4.59200	13.03500	4.18730	0.00(
NEW ZEALAND	0.57659	0.33530	4.41080	0.83190	7.90930	1.81090	103.65000	7.87790	12.96500	6.24050	0.00(
US	0.48511	0.23460	3.01520	0.33359	6.57130	0.55625	69.80400	2.78820	12.79300	2.82050	0.00(
JAPAN	0.96375	0.27056	17.37100	1.08060	8.14490	2.14040	93.12700	8.63610	13.50600	7.17250	0.00(
SINGAPORE	1.29400	0.48093	138.77000	3.54580	21.72900	5.12840	510.41000	35.17100	14.34300	9.08850	0.00(
HONG KONG	1.65700	0.82908	84.10400	8.95120	13.13000	4.65300	203.81000	26.53100	14.05600	9.28760	0.00(
CHINA	2.27630	1.30470	142.51000	43.92500	9.20840	7.76300	106.43000	70.08700	13.71700	9.75190	0.00(
INDONESIA	13.06600	1.91480	33440.00000	46.82900	18.99000	5.80980	381.27000	43.15400	14.23700	9.31770	0.00(
KOREA	3.02470	1.61020	146.63000	28.48000	8.85110	4.08380	106.41000	21.25900	13.49700	9.05240	0.00(
MALAYSIA	1.45670	1.63930	84.98100	85.65800	14.87200	8.05250	272.12000	74.05700	13.88900	9.88580	0.00(
PHILIPPINES	1.94280	1.35020	165.17000	101.43000	12.23900	10.60800	168.26000	117.68000	13.93600	10.07400	0.00(
TAIWAN	1.97410	0.84895	105.00000	8.62250	11.44900	4.19790	169.26000	22.78800	13.74900	8.83310	0.00(
THAILAND	3.66420	1.94100	318.85000	56.56100	8.12060	5.07520	77.56000	31.82300	13.71700	9.41940	0.00(

ARGENTINA	2.61990	0.82221	120.15000	6.05130	6.48700	3.72900	49.35400	20.76400	13.50500	8.28480	0.000
BRAZIL	2.73640	1.53740	101.37000	30.94900	6.97150	6.35960	62.62700	51.13300	13.36000	9.37700	0.000
CHILE	0.64915	0.35699	7.60390	1.23590	9.25950	2.31540	121.87000	10.17600	13.18300	7.00330	0.000
MEXICO	1.89920	0.61526	79.55400	2.23100	10.18200	2.36570	126.64000	11.67800	13.66300	6.85380	0.000
VENEZUELA	3.02680	1.74480	202.84000	39.72900	8.35710	5.72540	86.19100	43.68500	13.65500	9.29230	0.000
PORTUGAL	0.70840	0.32258	29.35800	0.88979	20.88000	2.29770	473.47000	11.75100	14.00800	6.64590	0.000
TURKEY	6.71140	4.40220	483.06000	353.82000	5.68210	7.34820	40.56800	64.10900	13.27000	9.73660	0.000
POLAND	3.20950	1.27750	181.45000	25.32400	7.85440	6.45510	79.36100	50.30200	13.53900	9.44450	0.000

TABLE 1B Summary statistics – nominally annualised return

	mean return		variance		skewness		kurtosis		W-statistic		Prob<	
	Weekly	Monthly	Weekly	Monthly	Weekly	Monthly	Weekly	Monthly	Weekly	Monthly	Weekly	M
AUSTRIA	0.06979	0.09971	0.58012	0.30105	-0.19678	-0.08261	3.19890	2.91810	1.47160	0.05615	0.07056	0
BELGIUM	0.07291	0.10553	0.70034	0.28524	-0.05076	-0.62508	5.31610	3.88080	5.57550	2.40340	0.00000	0
DENMARK	0.09545	0.13618	0.69807	0.33047	-0.55936	-0.51019	5.56810	3.81940	5.66630	1.62860	0.00000	0
FINLAND	0.15575	0.22246	2.55450	1.26990	-0.27073	0.29858	5.65480	4.11970	6.15200	1.86240	0.00000	0
FRANCE	0.07009	0.09992	0.78630	0.36445	-0.03060	-0.16191	4.50320	2.84990	4.30280	-0.02713	0.00001	0
GERMANY	0.05875	0.08383	0.90043	0.41970	-0.14429	-0.37730	5.18200	4.02180	5.61630	1.80640	0.00000	0
GREECE	0.10047	0.14593	1.93690	0.98588	0.32808	0.32069	5.29900	4.20200	6.23040	1.61190	0.00000	0
IRELAND	0.09748	0.13833	0.79417	0.37247	-0.41326	-0.48274	5.97230	3.86170	6.36790	2.18810	0.00000	0
ITALY	0.07152	0.10558	1.15370	0.59380	0.34395	0.49251	7.27480	3.14610	6.71200	1.53100	0.00000	0
NETHERLANDS	0.06658	0.09315	0.77091	0.32230	-0.24315	-0.79025	6.55160	3.82670	6.98470	3.21270	0.00000	0
NORWAY	0.08931	0.13003	1.00840	0.52538	-0.26367	-0.64022	6.00650	5.01580	6.45470	2.67390	0.00000	0
SPAIN	0.09264	0.13584	0.89519	0.45946	0.01148	0.04213	4.48370	4.07360	4.13530	0.70311	0.00002	0
SWEDEN	0.10429	0.14987	1.54790	0.74502	0.19587	-0.24478	7.55980	3.82980	6.56470	0.36564	0.00000	0
SWITZERLAND	0.07982	0.11475	0.70155	0.31785	0.07625	-0.39013	5.69970	3.99710	5.70010	1.10500	0.00000	0
UK	0.05467	0.07813	0.54750	0.21895	-0.10234	-0.40089	5.03370	2.8094	4.92480	1.692	0.00000	0.(
AUSTRALIA	0.06915	0.10165	0.67948	0.37597	-0.15739	-0.19811	4.02500	3.0163	2.74750	0.40807	0.00300	0
CANADA	0.08205	0.1206	0.74695	0.37958	-0.47999	-0.8191	5.60510	4.9721	6.38160	3.1047	0.00000	0.(
NEW ZEALAND	0.05914	0.088169	0.82815	0.49031	-0.28199	-0.34948	4.75820	3.4381	4.91760	0.93048	0.00000	0
US	0.07004	0.099154	0.67912	0.2772	-0.45562	-0.61545	6.68970	3.4874	6.53500	2.4264	0.00000	0.(
JAPAN	-0.00101	-0.002759	1.20020	0.52518	0.37556	0.31147	3.96260	2.7833	4.01580	1.2379	0.00003	0
SINGAPORE	0.03017	0.048651	1.23560	0.72424	-0.17660	0.17014	8.27150	4.9293	7.50840	3.8429	0.00000	6.(
HONG KONG	0.06180	0.098109	1.67890	1.0196	-0.13365	0.42398	5.49480	5.0792	5.49970	3.3389	0.00000	0.(
CHINA	0.12611	0.22589	4.94740	3.73520	8.50740	6.64160	132.83000	61.58900	12.31900	8.83640	0.00000	0
INDONESIA	0.04150	0.04473	6.56570	2.79250	0.89579	0.45864	18.39600	4.83720	10.12200	2.39920	0.00000	0
KOREA	0.07730	0.11768	4.49500	2.52350	-0.06209	1.28060	10.27200	8.33590	8.09720	4.74010	0.00000	0
MALAYSIA	0.01374	0.01978	2.83070	1.53960	0.49157	0.77794	20.25200	7.33090	10.54700	5.06020	0.00000	0
PHILIPPINES	-0.02129	-0.02825	2.06850	1.30930	-0.17835	0.91919	8.51820	7.87940	7.78850	4.26840	0.00000	0
TAIWAN	0.03217	0.04333	2.19940	1.20420	0.34699	0.42090	5.46090	3.65710	5.76190	1.58540	0.00000	0

THAILAND	-0.00275	0.00417	3.73140	2.20420	0.46284	0.45786	5.83060	4.38810	6.72100	3.15890	0.00000	0
ARGENTINA	0.01840	0.02933	2.76070	1.43720	-0.23701	-0.23066	6.48960	3.52210	6.33900	-0.33564	0.00000	0
BRAZIL	0.07791	0.12965	2.96240	1.88290	-0.35838	-0.08693	4.45570	4.47730	4.74480	2.27380	0.00000	0
CHILE	0.03523	0.05763	0.91838	0.57641	-0.13104	-0.16433	4.75260	4.26520	4.59810	1.52720	0.00000	0
MEXICO	0.05619	0.08256	2.22530	1.17340	-0.29475	-1.07380	6.92210	5.60560	6.64200	4.15550	0.00000	0
VENEZUELA	-0.00371	0.01978	4.00330	2.40730	-1.23730	-0.27303	19.66200	5.34300	9.67400	3.72950	0.00000	0
PORTUGAL	0.05898	0.08658	0.78903	0.44117	0.08022	0.02674	5.83830	3.11330	5.40000	-0.28934	0.00000	0
TURKEY	0.18519	0.27534	7.95330	4.61520	0.15600	0.84752	9.90150	5.46040	7.82340	3.79590	0.00000	0
POLAND	0.07048	0.09692	2.81630	1.54200	0.01553	0.21083	4.44980	4.62870	4.78670	1.98770	0.00000	0

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TABLE 2A
Polynomial goal programming: developed markets with annualised weekly returns

	Effective annualisation (EAR)		Nominal annualisation	
Parameters				
a	**1**	**1**	**1**	**1**
b	**0**	**1**	**0**	**1**
Optimal portfolio composition				
AUSTRIA	30.79%		19.93%	
BELGIUM				3.68%
DENMARK	28.55%		28.00%	
FINLAND			11.25%	5.57%
FRANCE				
GERMANY				
GREECE	0.89%		3.02%	
IRELAND			21.02%	
ITALY				47.57%
NETHERLANDS				
NORWAY				
SPAIN				
SWEDEN				
SWITZERLAND				27.10%
UK				
AUSTRALIA	12.22%		2.12%	
CANADA	4.71%		10.03%	
NEW ZEALAND	6.71%			
US	9.49%		4.64%	
JAPAN	6.41%			
SINGAPORE	0.23%	100.00%		
HONG KONG				16.08%
Optimal portfolio statistics (all are unit variance)				
mean	54.33%	10.99%	9.46%	7.70%
skewness	2.21	21.71	-0.12	0.27

Note: The weight in the goal programming model on deviation from maximum return is ***a***,
the weight on deviation from maximum skewness is ***b***.

TABLE 2B

Polynomial goal programming: developed markets with annualised monthly returns

	Effective annualisation (EAR)		Nominal annualisation	
Parameters				
a	**1**	**1**	**1**	**1**
b	**0**	**1**	**0**	**1**
Optimal portfolio composition				
AUSTRIA	9.29%		9.48%	2.87%
BELGIUM	2.19%		1.76%	
DENMARK	20.50%		33.97%	
FINLAND	1.64%		5.19%	
FRANCE				
GERMANY				
GREECE	4.52%	100.00%	0.50%	
IRELAND	8.94%		16.57%	
ITALY	3.82%			
NETHERLANDS				
NORWAY	0.82%			
SPAIN				
SWEDEN	3.61%			
SWITZERLAND				40.97%
UK				
AUSTRALIA	8.33%		4.77%	
CANADA				
NEW ZEALAND	6.79%			
US	26.78%		27.76%	
JAPAN	1.23%			
SINGAPORE	1.48%			
HONG KONG	0.04%			56.15%
Optimal portfolio statistics (all are unit variance)				
mean	63.87%	29.90%	12.51%	10.50%
skewness	0.78	5.57	-0.04	0.17

Note: The weight in the goal programming model on deviation from maximum return is ***a***,
the weight on deviation from maximum skewness is
b.

TABLE 3A

Polynomial goal programming: developing markets with annualised weekly returns

	Effective annualisation (EAR)		Nominal annualisation	
Parameters				
a	**1**	**1**	**1**	**1**
b	**0**	**1**	**0**	**1**
Optimal portfolio composition				
CHINA	12.89%	4.98%	22.01%	80.54%
INDONESIA		58.45%		
KOREA	9.49%		5.23%	
MALAYSIA	3.53%			0.97%
PHILIPPINES	4.40%			
TAIWAN	9.13%	32.75%		
THAILAND	2.44%			
ARGENTINA	7.32%			
BRAZIL	9.80%	0.51%	3.86%	2.91%
CHILE	8.12%		1.74%	3.24%
MEXICO	6.28%		2.60%	3.33%
VENEZUELA	8.60%			
PORTUGAL	5.00%		48.07%	7.86%
TURKEY	8.15%	3.32%	16.07%	
POLAND	4.85%		0.43%	1.15%
Optimal portfolio statistics (all are unit variance)				
mean	61.05%	8.02%	9.53%	11.24%
skewness	2.72	18.99	1.23	50.88

Note: The weight in the goal programming model on deviation from maximum return is ***a***,
the weight on deviation from maximum skewness
is ***b***.

TABLE 3B
Polynomial goal programming: developing markets with annualised monthly returns

	Effective annualisation (EAR)		Nominal annualisation	
Parameters				
a	**1**	**1**	**1**	**1**
b	**0**	**1**	**0**	**1**
Optimal portfolio composition				
CHINA	5.23%	1.76%	21.24%	90.28%
INDONESIA	4.19%			
KOREA	2.75%	0.46%	9.21%	
MALAYSIA	0.56%			
PHILIPPINES		97.78%		1.71%
TAIWAN	4.44%			
THAILAND				2.78%
ARGENTINA	8.36%			
BRAZIL	4.98%		4.82%	5.23%
CHILE	4.25%			
MEXICO	15.00%		2.75%	
VENEZUELA	6.49%			
PORTUGAL	37.31%		45.23%	
TURKEY	1.83%		16.74%	
POLAND	4.60%			
Optimal portfolio statistics (all are unit variance)				
mean	68.91%	13.70%	15.26%	21.04%
skewness	1.33	10.57	0.67	36.65

Note: The weight in the goal programming model on deviation from maximum return is ***a***,
the weight on deviation from maximum skewness
is ***b***.

TABLE 4A

Polynomial goal programming: all markets with annualised weekly returns

	Effective annualisation (EAR)		Nominal annualisation	
Parameters				
a	**1**	**1**	**1**	**1**
b	**0**	**1**	**0**	**1**
Optimal portfolio composition				
AUSTRIA	29.67%		25.18%	
BELGIUM				
DENMARK	*22.38%*		23.52%	
FINLAND	*3.31%*		4.14%	
FRANCE				
GERMANY				
GREECE	*1.91%*		1.67%	
IRELAND	*0.08%*			56.30%
ITALY	*12.63%*		16.78%	
NETHERLANDS		99.99%		
NORWAY				12.67%
SPAIN				31.03%
SWEDEN				
SWITZERLAND				
UK				
AUSTRALIA	2.23%		8.19%	
CANADA	1.19%		0.30%	
NEW ZEALAND			4.70%	
US	4.26%		6.18%	
JAPAN	3.92%		4.52%	
SINGAPORE			1.60%	
HONG KONG				
CHINA	3.25%	0.01%	3.21%	
INDONESIA				
KOREA	1.49%			
MALAYSIA	1.52%			
PHILIPPINES	1.32%			
TAIWAN	1.99%			
THAILAND				
ARGENTINA	1.61%			
BRAZIL	1.68%			
CHILE				
MEXICO	0.83%			
VENEZUELA	2.06%			
PORTUGAL				
TURKEY	1.85%			
POLAND	0.82%			
Optimal portfolio statistics (all are unit variance)				
mean	74.62%	12.25%	59.69%	70.66%
skewness	1.79	20.38	1.71	972.88

Note: The weight in the goal programming model on deviation from maximum return is ***a***,

the weight on deviation from maximum skewness is
b.

TABLE 4B

Polynomial goal programming: all markets with annualised monthly returns

	Effective annualisation (EAR)		Nominal annualisation	
Parameters				
a	**1**	**1**	**1**	**1**
b	**0**	**1**	**0**	**1**
Optimal portfolio composition				
AUSTRIA	6.75%		4.31%	
BELGIUM	*0.86%*			
DENMARK	*25.83%*		20.61%	
FINLAND	*0.15%*		0.78%	
FRANCE				
GERMANY				
GREECE	*4.79%*	2.21%	4.69%	
IRELAND	*18.89%*		9.90%	
ITALY				
NETHERLANDS				
NORWAY			1.81%	
SPAIN				
SWEDEN	*5.07%*		9.69%	
SWITZERLAND				39.47%
UK				
AUSTRALIA	13.85%		11.79%	
CANADA				
NEW ZEALAND			7.73%	
US	3.07%		21.02%	
JAPAN		3.66%	2.49%	
SINGAPORE				
HONG KONG				60.53%
CHINA	2.89%	1.72%	5.18%	
INDONESIA	1.86%			
KOREA	1.12%			
MALAYSIA				
PHILIPPINES		92.41%		
TAIWAN	2.76%			
THAILAND				
ARGENTINA	3.29%			
BRAZIL	3.44%			
CHILE				
MEXICO	0.79%			
VENEZUELA	1.89%			
PORTUGAL				
TURKEY	0.95%			
POLAND	1.75%			
Optimal portfolio statistics (all are unit variance)				
mean	56.84%	129.91%	33.59%	50.09%
skewness	0.28	8548	0.07	23.33

Note: The weight in the goal programming model on deviation from maximum return is ***a***,

the weight on deviation from maximum skewness is
b.

Endnotes:

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[1] The nominal annualization will not have any effect on the variance and skewness of the rates of returns because nominal annualization just changes the scale of the distribution whereas effective annualized rate will change the variance and skewness. We define

$$\text{EAR} = \left(1 + \textit{periodic rate}\right)^{t} - 1$$

[2] The explanation from the start of the section to equation (9) has been generously borrowed from Prakash, de Boyrie, Hamid, and Smyser (1997). In fact, we use the same symbols as have been used by them to maintain continuity.

[3] Intervaling is defined as the length between two data points over which the returns are measured, e.g., a day, a week, a month etc.

[4] At what time the process terminates is of no consequence. It can continue indefinitely. The process can either be continuous or discrete. However any continuous process can be approximated by infinitesimally small discrete jumps.

[5] As mentioned earlier, we refrain from providing the literature review as this has been covered exhaustively in the papers by Chunhachinda et al. (1997) and Prakash et al. (2003). Furthermore, since we are using exactly the same procedure as Chunhachinda et al. (1997), we refrain from repeating the same as well.

[6] Results from other combination values of *a* and *b* are not reported in the paper but are available to readers upon request.