Spicing Up a Portfolio with Commodity Futures: *Still a Good Recipe?*

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Empirical portfolio research using commodity futures often reports negative correlations between commodities and equity markets, a feature highly valued in the areas of portfolio diversification and optimization.[1] Consequently, adding commodity futures to a portfolio of stocks can generate significant diversification benefits, as well as *potentially* reduced risk and increased returns.[2] The diversification and performance benefits of commodity markets have enticed investors to invest billions of dollars in this asset class during the past decade, despite the high degree of volatility experienced by certain individual commodity futures. However, the recent surge in commodity investment has also caused the correlation between equity markets and commodity futures to increase (Tang and Xiong [2012], You and Daigler [2013]), and therefore one can question whether commodities' long sought-after low correlation and large positive return properties are still able to deliver tangible value.

This article provides an extended examination of the ability of individual futures contracts to enhance the risk and return of a portfolio as an *investment* vehicle over the past two decades. We extend previous examinations of futures to enhance a portfolio by analyzing factors typically ignored—namely correlation levels by the type of futures—employing targeted risk levels in combination with optimal portfolio models,[3] using rebalancing strategies to generate portfolio risk and return values, determining the effect of tail risk on the portfolio results, and analyzing more recent data. More specifically, we examine the benefits of holding *individual* commodity futures in one's investment portfolio during recent market conditions that include both the market and commodity booms of the 1990s and 2000s, respectively, as well as the market declines owing to the technology and financial crashes of 2000–2002 and 2008. Our results show how a portfolio of individual commodity and equity futures is able to outperform an equity portfolio in terms of risk and return, during both equity and commodity bull and bear market periods.

We find that the correlation between equity and commodity markets is often still low, although this correlation has increased during recent years. More importantly, we also find that including commodity futures in one's investment portfolio improves portfolio performance *not only* on an ex ante, but also on an ex post basis, with such portfolios consistently outperforming equity-only portfolios. Moreover, our results are robust to the optimization of the portfolio at different rebalancing frequencies, targeted risk levels, and time periods. Changes in the portfolio weights support a higher rebalancing frequency, with the weights being relatively stable over time within categories of contracts.

Additionally, the resultant optimized portfolios exhibit lower tail risk than the benchmark equity indexes do. Consequently, our comprehensive results support the continuing benefits of using commodity futures as an investment strategy.

EVIDENCE AND CONSIDERATIONS FOR ADDING COMMODITY FUTURES CONTRACTS TO A PORTFOLIO

Past efforts employing commodity futures as part of an investment portfolio generally have found low (or even negative) correlations between equity and commodity markets, providing preliminary evidence of potential diversification benefits.[4] In the context of portfolio construction and optimization, studies examining the risk and return benefits of incorporating individual commodities are scarce. In fact, most such research focuses on adding a commodity *index* of futures to an investment portfolio rather than adding individual futures contracts. Specifically, Bodie and Rosansky [1980], Greer [1994], and Conover et al. [2010] find that adding a commodity index to an all-equity portfolio or equity index leads to a combined portfolio with a lower standard deviation. Satyanarayan and Varangis [1996] show that adding the Goldman Sachs Commodity Index to an international equity portfolio can improve the return of the portfolio for any given level of risk. Georgiev [2001] determines that a direct investment in the GSCI can provide downside portfolio protection. Anson [1998] finds that the portfolio's Sharpe ratio is improved when nonfinancial futures contracts are added to an even already well-diversified portfolio of stocks and bonds. Ankrim and Hensel [1993]; Jensen, Johnson, and Mercer [2000]; and Idzorek [2007] reach similar conclusions using different sample periods and a mean–variance optimization model. However, the use of a commodity index creates several issues: first, a commodity index typically is biased toward certain types of futures; for example, the GSCI is heavily weighted toward energy futures. Second, diversifying using a commodities futures index ignores the potential benefit of the low correlations between specific individual/groups of futures contracts. Finally, employing only indexes mitigates the benefits provided from determining the optimal weights generated from the Markowitz model.

Several recent studies do examine the benefit of diversifying with individual futures. You and Daigler [2010] conclude that randomly adding individual futures contracts to a portfolio can effectively diversify away the higher-moment risk of the portfolio. You and Daigler [2013] find that, on an ex ante basis, an optimal portfolio of commodity futures outperforms the S&P 500 Index as well as individual futures, and that the ex post results of the same optimal portfolios outperform a *naïve* portfolio of equal weightings. However, our current article considers a longer dataset, target risk levels, the rebalancing effect, and the frequency of rebalancing.[5]

One special characteristic of commodities is that they offer a fairly unique hedge, that is, one against inflation, since commodity prices usually rise when inflation is increasing. Most asset classes do not benefit from a rising inflation, but commodities usually do. The risk of high inflation is what can lead market traders and investors to move away from equities. In such an environment, the central bank usually tries to keep inflationary pressure under control by raising interest rates, hoping to get investors to move to fixed-income instruments as a way to decrease the excess liquidity in the system. In theory, lower liquidity levels should reduce speculative demand for goods in the economy and thus slow down the overall increase in prices. The potential for future higher interest rates is usually bad news for the equity market, because it encourages investors to lock in their cash from risky stocks to more attractive and less risky securities such as bonds and money market instruments. As the demand for stocks goes down, share prices tend to decrease. Another way to look at it is from the point of view of valuation whereby higher expected future inflation increases the risk premium on equities, leading to higher required rates of return and thus to lower stock prices. Commodities will therefore often display low or negative correlations with respect to traditional equities.

Additionally, some asset classes (such as bonds, for instance) display low correlation levels with respect to traditional equities in the long run but can suffer from much higher correlation levels in the short term during periods of crises, implying that the diversification benefit disappears when it is most needed. This isn't necessarily the case with commodities, as their short-term correlation levels with respect to equities can remain low even in the short term. As an illustrative example, the iShares S&P GSCI Commodity-Indexed Trust (GSG) has since its inception (July 21, 2006) shown a correlation of 0.44 with respect to the S&P 500 (thus in the long run) and yet managed to maintain a correlation of 0.40 during 2008 and even a slightly-higher-only correlation of 0.58 during the months of September and October of 2008 (the worst of the crisis).

Our article improves upon previous studies by employing all of the actively traded U.S. futures contracts and considering various issues associated with portfolio construction, such as a more recent and longer time period, bull and bear market distinctions, the frequency of rebalancing, changes in correlations, and the effect of tail risk (extreme losses).

DATA AND METHODOLOGY

Data

We obtain daily futures prices from the Commodity Research Bureau (CRB). The choice of using daily data is a deliberate one: it provides a larger dataset for a given time period than weekly or monthly data, which allows a more precise targeting of a specific standard deviation and risk level. Our futures sample includes all actively traded nearby contracts in U.S. markets from 1990 through 2012.[6,7] We include the following 16 financial futures in the analysis: five equity index futures, four interest rate futures, and seven currency futures. The following 21 commodity futures employed in our analysis are: three metal futures, four energy contracts, six grain futures, five subtropical contracts (cocoa, coffee, sugar, orange juice, cotton), and three live cattle futures. Our dataset also includes two futures contracts on commodity indexes: the GSCI and the CRB.[8] In addition to creating optimized portfolios, we also construct a naïve futures portfolio employing an equal weighting of all 39 futures contracts, as well as a naïve commodity portfolio made up of an equal weighting of the commodity-only futures contracts. Finally, we also construct a weighted-average index for each type of futures category in order to better examine the different asset groups. We calculate all Sharpe ratios in this article using the daily risk-free rate reported by the Federal Reserve Bank of St. Louis.

Methodology

We compose the desired portfolios by adopting a straightforward two-moment risk–return framework in order to identify the Markowitz mean–variance optimal allocations of futures contracts. The effect of other moments is discussed in the results. For a portfolio P with n assets, the portfolio's return μ_p and risk σ_p^2 characteristics are calculated as:

$$\mu_p = \sum_{i=1}^{n} \mu_i x_i$$

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{i,j} x_i x_j \qquad (1)$$

Subject to

$$\sum_{i=1}^{n} x_i = 1$$

$$x_i \geq 0,\ i = 1, 2, \ldots n$$

where μ_i, σ_i, and x_i are the mean, standard deviation, and weight of the ith asset in the portfolio. The well-known solution to the Markowitz procedure maximizes the return given a fixed level of risk, or minimizes the risk given a fixed targeted return. We implement the procedure by composing portfolios with a given chosen risk level and therefore solve for the weights that maximize the Sharpe ratio, given the targeted standard deviations of 5%, 10%, 15%, and 20%. Additionally, since imposing restrictive constraints typically benefits out-of-sample results (Jagannathan and Ma [2003], DeMiguel et al. [2009]), we do not allow short sales. We also enforce a maximum allocation of 10% in any given futures contract to minimize the effect of any futures with a jump component; moreover, this allocation is consistent with industry practices and the fact that constraining allocations in-sample, while reducing the Sharpe ratio in-sample, produces superior results out-of-sample in the long run. Finally, we re-optimize our futures portfolios every three, six, and twelve months.

We then examine the portfolios' out-of-sample performance over different time periods.[9] In order to minimize any potential data mining bias, we adopt the straightforward approach of setting the in-sample and out-of-sample periods equal in length. Thus, within any period studied, a portfolio is re-optimized when its out-of-sample investment period reaches the length of the in-sample data period used to determine the portfolio weights. The in- and out-of-sample periods are unique in that they do not overlap in time, providing a clear analysis of the returns and risks of the period. In a nutshell, at a given point in time when a portfolio is to be rebalanced, the length of the window of time chosen to estimate the parameters (used to construct the portfolio) is set to be the same as the length of the out-of-sample window during which the portfolio remains untouched (the out-of-sample horizon/holding period). This sensible straightforward methodology is meant to minimize biases associated with data mining and parameter overfitting possibly associated with testing every possible in-sample and out-of-sample length combinations and selecting the one that performs the best over the testing period (but perhaps not in the future, in a similar fashion that an overfitted regression might describe things well in sample but possibly quite poorly out of sample).

For illustrative purposes, at any given point in time when a portfolio needs to be re-optimized, if the portfolio is afterward to remain untouched for, say, six months, exactly six months of recent historical daily returns are used to estimate the parameters and construct the portfolio. The performance of the portfolio over the out-of-sample horizon is then recorded for six months and "stored," until the portfolio is re-optimized again. The performance of the next portfolio is also recorded over its out-of-sample horizon, and so on. The multiple out-of-sample periods of performance are at the end of the backtest "glued" together to form a time series of out-of-sample daily returns, representative of how the strategy would actually have performed in practice over several years. Then, out-of-sample returns and risk levels can be analyzed in a variety of ways, and at any frequency (daily, weekly, monthly, or annually).

EMPIRICAL RESULTS

Statistics

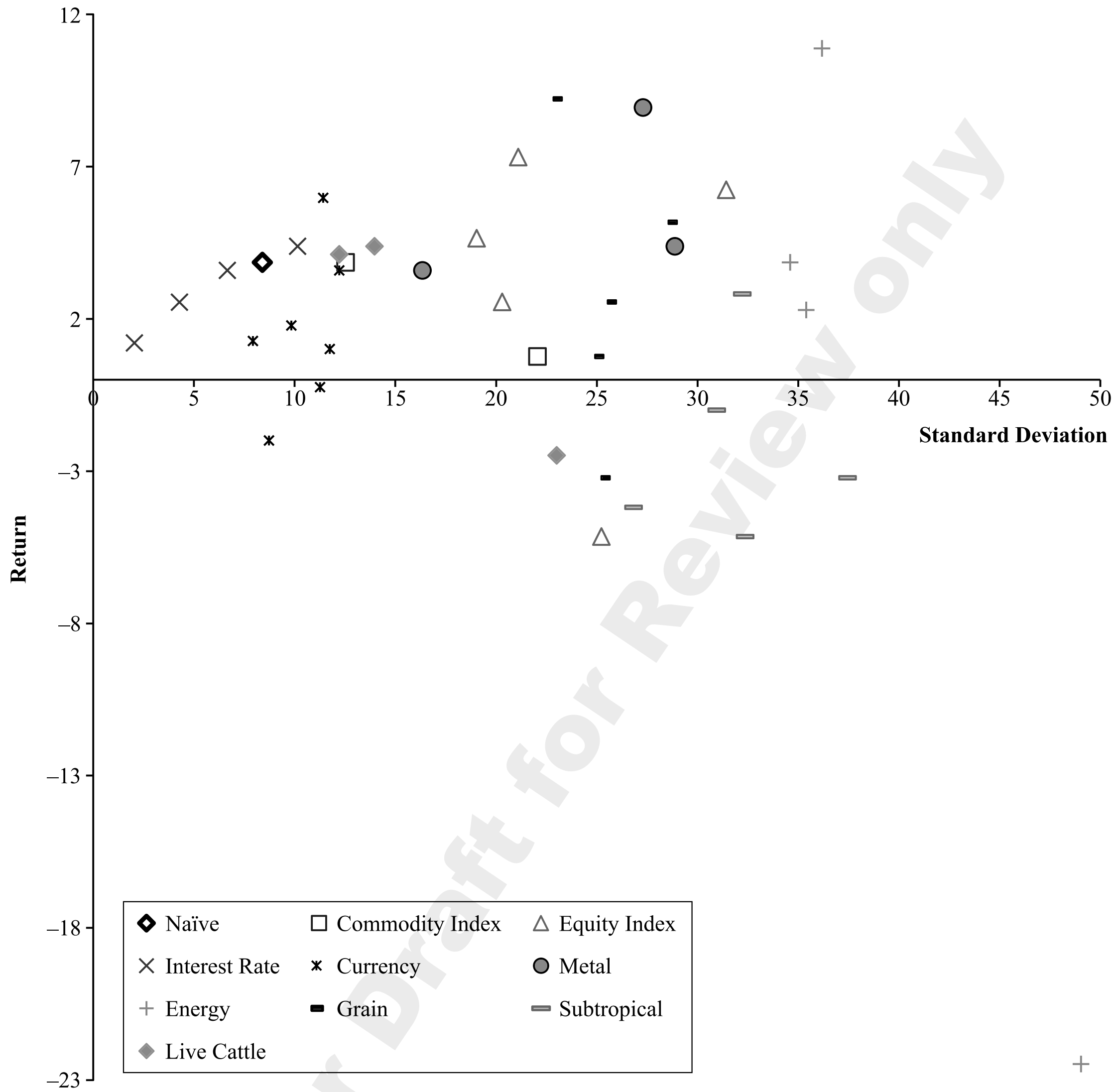
Exhibit 1 reports the annualized returns and characteristics of all futures contracts during the time of this study.[10] Commodity futures generally display high standard deviations as well as highly variable cross-sectional returns. Gasoline futures generate the highest returns, followed by copper and soybean meal futures. Natural gas futures possess the highest standard deviation, followed by coffee and gasoline. Stock indexes generally show relatively high returns and high standard deviations, whereas interest rates and currency futures typically exhibit a lower risk–return profile. The futures contracts as a whole show a fairly narrow range of skewness and kurtosis values; however, this is attributable to the

EXHIBIT 1
Summary Annualized Statistics

	Arithmetic Mean (%)	Geometric Mean (%)	Stdev (%)	Skewness	Kurtosis
Naïve	4.03	3.85	8.41	–0.02	3.02
GSCI	3.02	0.76	22.07	–0.01	3.01
CRB	4.54	3.85	12.54	–0.02	3.02
Dow Jones	4.54	2.55	20.32	0.02	3.04
S&P 500	6.30	4.64	19.05	0.01	3.04
NASDAQ Stock Index	10.84	6.23	31.43	0.01	3.02
Midcap 400	9.32	7.31	21.11	–0.01	3.03
Nikkei Stock Index	–2.02	–5.15	25.24	0.01	3.02
2-Year Notes	1.21	1.20	2.06	–0.04	3.04
5-Year Notes	2.60	2.54	4.29	–0.01	3.01
10-Year Notes	3.78	3.59	6.67	–0.01	3.01
30-Year Bond	4.79	4.38	10.16	–0.01	3.01
Australian dollar	4.28	3.59	12.22	–0.03	3.04
British pound	2.27	1.78	9.84	–0.01	3.01
Canadian dollar	1.51	1.27	7.94	0.00	3.03
Mexican peso	6.30	5.97	11.43	–0.06	3.07
Japanese yen	0.25	–0.25	11.27	0.04	3.03
Swiss franc	1.76	1.01	11.75	0.00	3.02
U.S. dollar	–1.51	–2.00	8.73	0.00	3.01
Copper	12.35	8.94	27.30	0.00	3.02
Gold	5.04	3.59	16.35	0.00	3.03
Silver	8.32	4.38	28.89	–0.04	3.03
Crude Oil	8.57	2.29	35.40	–0.04	3.04
Gasoline	16.88	10.88	36.19	–0.02	3.02
Heating Oil	9.83	3.85	34.61	–0.03	3.04
Natural Gas	–13.36	–22.48	49.05	0.01	3.01
Cocoa	3.78	–1.00	30.96	0.01	3.01
Coffee	3.53	–3.22	37.46	0.01	3.01
Corn	0.00	–3.22	25.24	0.01	3.01
Cotton	–1.01	–4.19	26.83	–0.01	3.01
KC Wheat	5.80	2.55	25.56	0.00	3.01
Wheat	–2.77	–6.58	28.57	0.01	3.01
Orange Juice	0.00	–5.15	32.38	0.01	3.01
Soybeans	7.56	5.17	22.86	0.03	3.03
Soybean Meal	11.84	9.22	24.92	0.01	3.01
Soybean Oil	3.28	0.76	23.02	0.06	3.06
Sugar	8.06	2.81	32.23	–0.01	3.01
Feeder Cattle	4.79	4.11	12.22	–0.02	3.02
Lean Hogs	0.25	–2.49	23.02	0.00	3.01
Live Cattle	5.29	4.38	13.97	–0.01	3.01
Average	4.15	1.48	21.41	0.00	3.02

Exhibit 2

Annualized Risk–Return Graph of Individual Futures Contracts and the Naïve Portfolio



annualization effect discussed in the footnote. The naïve futures portfolio displays an average return and a relatively low level of risk compared with the individual futures contracts. The naïve portfolio's true (geometric) return (3.85%) is marginally lower than that of the S&P 500 Index (4.64%), whereas its standard deviation (8.41%) is much less than the standard deviation of the S&P 500 (19.05%). Consequently, the naïve portfolio exposes the investor to 2.18 units of risk per unit of return and the S&P 500 subjects an investor to 4.11 units of risk per unit of return. Therefore, the naïve portfolio offers a superior risk–return tradeoff compared with the S&P 500. Compared with the average geometric return and standard deviation of all individual futures (1.48% and 21.41%, respectively), even a naïve portfolio increases the return by 2.37% and reduces the risk by 13%.

Exhibit 2 plots the returns and standard deviations of individual futures contracts by category, as well as the return and standard deviation of the naïve portfolio. The interest rate group has the lowest return and standard deviation, followed by currency futures, and then live cattle, stock indexes, grain, and subtropical futures. The energy group has the highest return as well as the highest standard deviation, with the exception of natural gas futures that generate the worst performance of all futures with the lowest return and the highest standard deviation.

Correlations between Futures Categories

Exhibit 3 presents the Pearson correlations between and within the various categories of futures contracts. The correlations between each group pair are calculated as the average correlation between each pair of individual futures within those two groups. The within-group correlations are calculated as the average correlation between each pair of futures contracts within the same group. The within-group correlations show along the diagonal of Exhibit 3 and the between-group correlations are above the diagonal. Except for subtropical futures, the within-group correlations are generally higher than the between-groups correlations. One explanation for this is that we follow the common practice of classifying them (cocoa, coffee, sugar, orange juice, cotton) as one sector since they grow in subtropical (and tropical) areas. But they do not have as close a link to each other as other groups of commodity futures do. For example, the grain group contains corn, wheat, KC wheat, soybeans, soybean meal, and soybean oil: they are food for human and animal consumption and can substitute for each other. The cattle group contains three commodities: feeder cattle, lean hogs, and live cattle: they are meat, also for human consumption, and can substitute for each other too. The components in the energy and metal groups all have close links to other components within each group. Therefore, the within-group correlations are usually high for those groups, but not as much for the tropical group. The interest rate group is also negatively correlated with the other groups, proving that it is a useful investment vehicle for diversification and portfolio optimization purposes. The generally low correlations (ranging from −0.25 to 0.18) between the different futures groups show that investing in these different types of contracts, even naively, should significantly reduce the overall risk of the portfolio.

Since correlations are known to change over time, we also calculate rolling-window correlations in an effort to capture such changes. The correlations in Exhibit 4 show the one-year rolling window correlations between the S&P 500 Index returns and the futures group returns obtained by averaging the returns in each group.[11] The S&P 500 generally shows fairly low correlation levels with the other categories. The interest rate futures group displays a steadily decreasing correlation with the S&P 500, with that correlation becoming more negative in the last two market downturns. Therefore, interest rate futures also appear to be a useful diversification tool when combined with traditional equity positions, particularly during bear markets. However, the correlations between equity futures and currency futures, as well as between equity futures and commodity futures, have increased sharply since the financial crisis of 2008 (although they did not change substantially during the dotcom bubble and the crash of 2000–2002). Tang and Xiong [2012] argue that this increase is owing to the financialization of the commodity markets. Whether this upward shift is temporary or permanent remains to be seen.

EXHIBIT 3
Correlations between and within Different Groups of Futures Contracts

	SIF	Interest	Currency	Metal	Energy	Grain	Subtropical	Live Cattle
SIF	0.75	–0.20	0.10	0.08	0.05	0.10	0.07	0.06
Interest		0.65	–0.01	–0.03	–0.05	–0.07	–0.05	–0.05
Currency			0.03	0.15	0.06	0.08	0.06	0.04
Metal				0.45	0.15	0.18	0.12	0.08
Energy					0.48	0.11	0.06	0.05
Grain						0.57	0.13	–0.25
Subtropical							0.09	0.04
Live Cattle								0.42

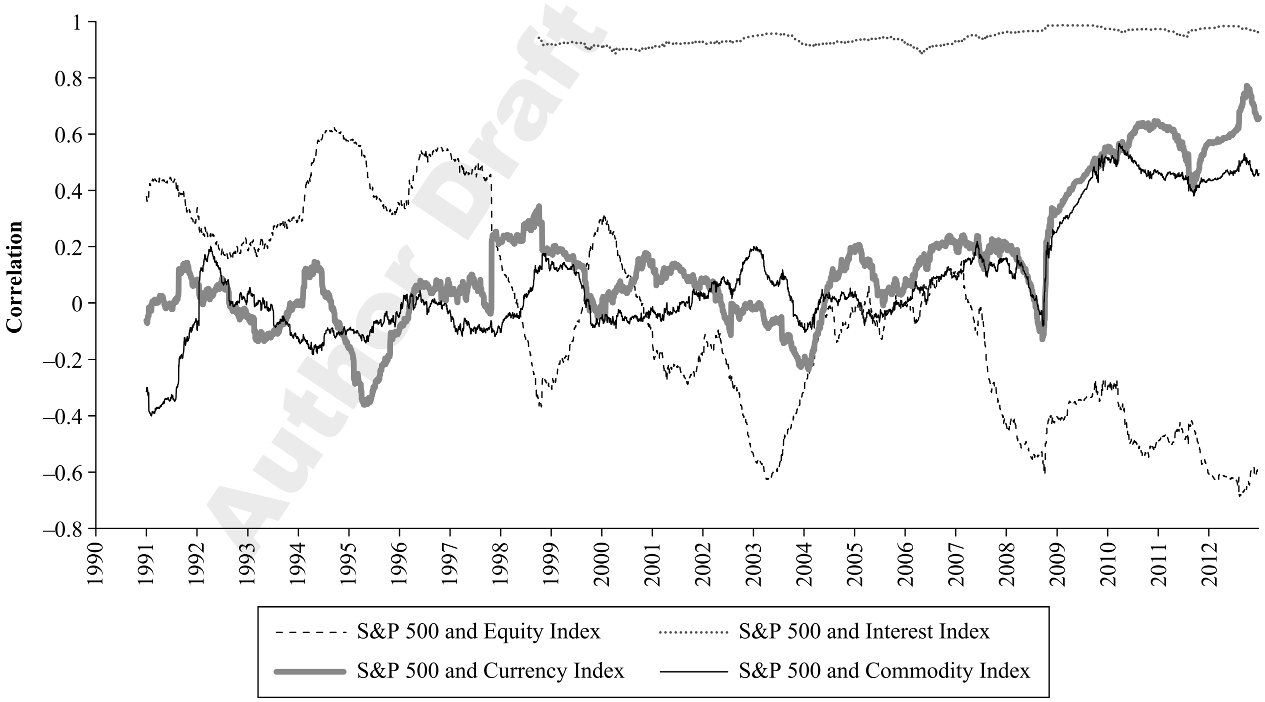
Notes: Correlations within each group are along the diagonal line and represent average values of all pairs of correlations between contracts within the same group. The between-groups correlations are off the diagonal line and are obtained by averaging all pairs of correlations between futures from the different pairwise groups. The subtropical group includes cocoa, coffee, sugar, orange juice, and cotton contracts.

OUT-OF-SAMPLE OPTIMAL PORTFOLIOS

The low correlations between the different futures contracts categories are consistent with significant potential diversification and mean–variance optimization benefits. In order to explore and quantify these benefits further, we optimize portfolios of commodity and noncommodity futures in a mean–variance setting using standard deviations of 5%, 10%, 15%, and 20%. We then rebalance these portfolios quarterly, semiannually, and annually for three different time periods (2000–2012, 1995–2012, 1991–2012). As previously discussed, the length of the in-sample period used to construct the portfolio is set equal to the out-of-sample horizon rebalancing frequency.[12] We report these out-of-sample results in Exhibit 5, together with the performance of several benchmark equity indexes (S&P 500, NASDAQ, and the Russell 2000). We measure the reported out-of-sample returns as geometric returns in order to reflect the true rate of return an investor would experience with the different strategies.[13]

Exhibit 5 reveals that portfolios rebalanced quarterly generally outperform (with a higher Sharpe ratio) portfolios rebalanced semiannually, which in turn outperform portfolios rebalanced annually. This finding is consistent with the benefit resulting from the rebalancing effect shown by Willenbrock [2011], although our "rebalancing" is much more than a mere naïve resetting of the weights, since we actually conduct a new mean–variance optimization at each time of rebalancing. From a Sharpe ratio point of view, the mean–variance portfolios outperform the equity indexes on an out-of-sample basis in most cases. Using three different market periods, 1991–2012 (without all commodity futures contracts available in the early years), 1995–2012 (with most futures contracts available for the entire period), and 2000–2012 (with all futures contracts available for the entire period), along with four possible targeted volatility levels (5%, 10%, 15%, and 20%) covering three different rebalancing frequencies (three months, six months, and one year), out of a total of 108 scenarios, the equity-commodity-optimized portfolios outperformed the S&P 500 Index on a risk-adjusted basis a

EXHIBIT 4
One-Year Rolling-Window Correlations between the S&P 500 and the Futures Groups



Notes: Several equity indexes did not start trading until the mid- or late 1990s. Therefore, our equity group index starts in the late 1990s.

EXHIBIT 5

Summary of Portfolios' Performance with Varying Rebalancing Frequencies and Targeted Standard Deviations

		2000–2012			1995–2012			1991–2012		
Rebalancing Frequency	**Portfolio**	**Return (%)**	**Stdev (%)**	**Sharpe Ratio**	**Return (%)**	**Stdev (%)**	**Sharpe Ratio**	**Return (%)**	**Stdev (%)**	**Sharpe Ratio**
3 months	Portfolio (20% Stdev)	9.98	19.17	0.41	9.67	17.81	0.38	7.86	16.84	0.28
	Portfolio (15% Stdev)	10.19	15.72	0.51	8.79	14.93	0.39	7.39	14.32	0.29
	Portfolio (10% Stdev)	8.64	11.49	0.56	9.59	11.12	0.59	7.39	10.74	0.39
	Portfolio (5% Stdev)	6.27	6.62	**0.62**	7.54	6.52	**0.70**	5.64	6.20	**0.39**
	S&P 500	–0.20	21.43	–0.11	6.48	20.05	0.17	6.90	18.71	0.20
	Russell 2000	4.22	26.61	0.08	7.11	23.75	0.17	8.89	21.93	0.26
	NASDAQ	–2.21	28.60	–0.15	8.05	26.71	0.19	9.99	24.77	0.27
6 months	Portfolio (20% Stdev)	6.18	19.93	0.20	8.20	18.77	0.28	5.85	17.50	0.15
	Portfolio (15% Stdev)	6.95	16.35	0.29	8.00	15.72	0.32	5.95	14.91	0.19
	Portfolio (10% Stdev)	5.75	11.27	0.32	7.72	11.02	0.43	5.85	10.81	0.26
	Portfolio (5% Stdev)	5.17	6.10	**0.49**	5.78	6.00	**0.47**	4.40	5.90	0.21
	S&P 500	–0.20	21.43	–0.11	6.48	20.05	0.17	6.90	18.71	0.20
	Russell 2000	4.22	26.61	0.08	7.11	23.75	0.17	8.89	21.93	0.26
	NASDAQ	–2.21	28.60	–0.15	8.05	26.71	0.19	9.99	24.77	**0.27**
1 year	Portfolio (20% Stdev)	5.87	20.32	0.18	7.70	19.06	0.25	6.02	18.17	0.16
	Portfolio (15% Stdev)	6.33	16.01	0.26	7.32	15.36	**0.28**	5.04	14.79	0.13
	Portfolio (10% Stdev)	6.24	11.26	**0.36**	5.41	11.12	0.22	2.67	10.82	–0.05
	Portfolio (5% Stdev)	3.30	5.85	0.20	3.61	5.94	0.11	2.34	5.58	–0.15
	S&P 500	–0.20	21.43	–0.11	6.48	20.05	0.17	6.90	18.71	0.20
	Russell 2000	4.22	26.61	0.08	7.11	23.75	0.17	8.89	21.93	0.26
	NASDAQ	–2.21	28.60	–0.15	8.05	26.71	0.19	9.99	24.77	**0.27**

Notes: This exhibit reports the out-of-sample results for the optimized portfolios with a 3-, 6-, and 12-month rebalancing frequency with a 5%, 10%, 15%, and 20% targeted standard deviation level for the three different sample periods. Three equity indexes from the same sample period are also reported for comparison. The returns reported in this article are the true (i.e., geometric) returns. Sharpe ratios can be negative if the portfolio return is lower than the risk-free rate.

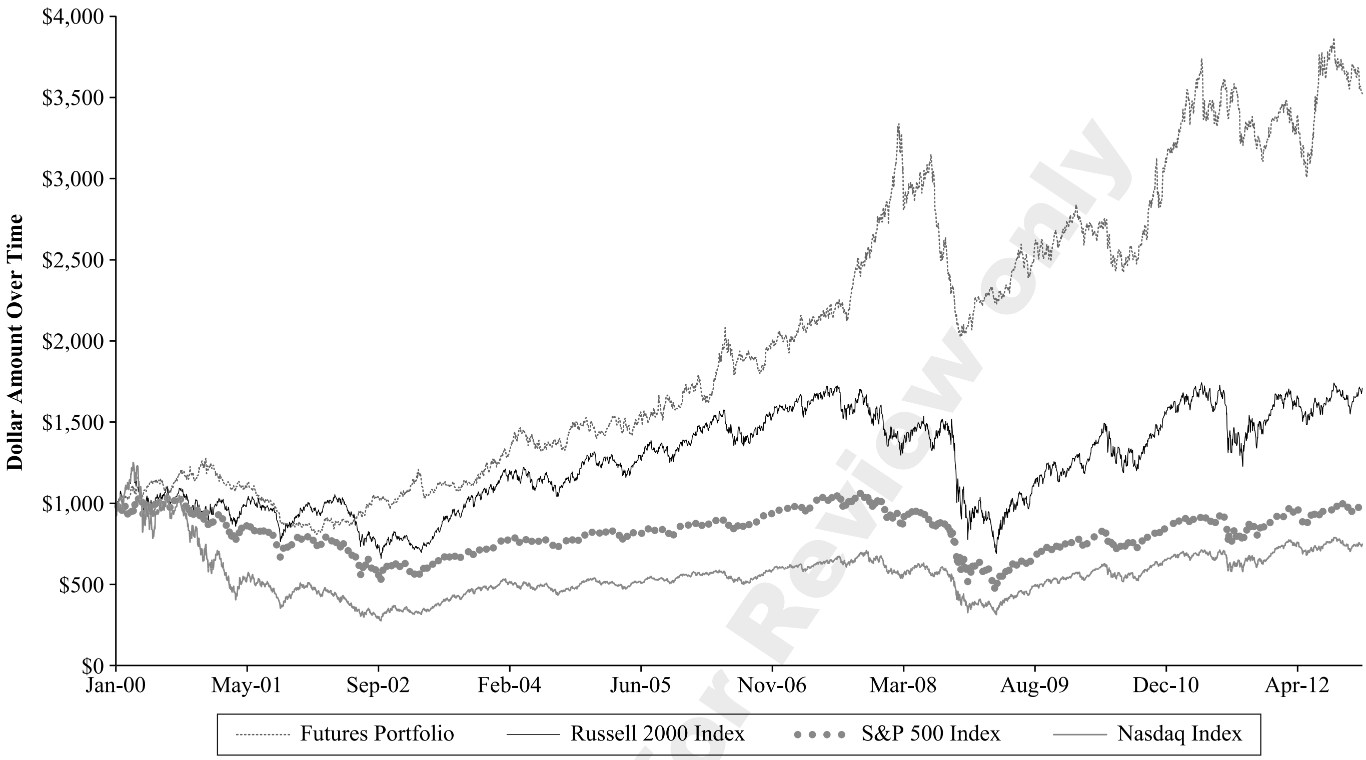
total of 101 times. As an example, when targeting a 5% standard deviation level, the portfolios rebalanced at a three-month frequency produce a Sharpe ratio of 0.62, 0.70, and 0.39 for the three sample periods, respectively. These Sharpe ratios are significantly higher than those of the three equity indexes (−0.11, 0.17, and 0.20 for the equivalent periods).

Additionally, if we compare the average performance of all individual futures contracts (as given in Exhibit 1) to the mean–variance portfolio over the same sample period (rebalanced at three-month, six-month, and one-year intervals, with a targeted standard deviation of 20% for the 1991–2012 period in Exhibit 5), we observe a lower risk profile for the three optimized portfolios (16.84%, 17.50%, and 18.17%, respectively, for the three mean–variance portfolios, as compared with 21.41% for the average for all futures over the same period). Moreover, the average individual futures contract only yields a *geometric mean return* of 1.48% (Exhibit 1), whereas the mean–variance portfolios produce geometric mean returns of 7.86%, 5.85%, and 6.02%, respectively, generating a substantial "optimization alpha," averaging a 5.1% excess return across the three portfolios.[14]

Futures contracts also have trading advantages over cash markets and physical commodities, especially in terms of liquidity and cost. Transaction costs for cash investments can be significant, while futures can instead be bought on margin, often at only about 5% of the total value of the contract. Margin accounts are marked to market and only have to be resupplied if the trader's margin falls short of requirements. The March 2015 CME

Exhibit 6

Cumulative Price Appreciation of Mean–Variance Portfolios versus Equity Indexes



Note: This exhibit shows the time path (2000–2012) of $1,000 invested in several equity indexes and the optimal portfolio of futures contracts at a 15% targeted standard deviation, rebalanced every three months.

Group white paper[15] shows how a fully-funded investor using ETFs will achieve an annual cost advantage of approximately 12 basis points over futures contracts, but the advantage disappears for shorter holding periods and the round-trip cost is actually the same for a four-month horizon. Additionally, when compared on a leveraged basis (since futures contracts are leveraged positions by nature), the benefit of futures contracts over ETFs is even more pronounced as an investor achieves a cost advantage of between 8 and 23 basis points for 2x and 8x leveraged positions, respectively, when using futures contracts rather than ETFs.

PORTFOLIO GROWTH AND STABILITY

A mean–variance optimized portfolio is not necessarily able to match the performance of a pure equity strategy during times of rapidly rising stock market prices, such as those of the 1990s. However, such a risk-controlled futures portfolio is better able to protect an investor against market downturns and therefore is able to grow more consistently and steadily during an entire market cycle. In order to illustrate this point, Exhibit 6 plots the mean–variance futures portfolio dollar value evolution from 2000 to 2012 for a quarterly re-optimized portfolio targeting a 15% standard deviation relative to the value of the S&P 500, Russell 2000, and NASDAQ indexes.[16] Exhibit 6 shows that a portfolio made up of the S&P 500 Index with a hypothetical $1,000 starting value actually *falls* to $974 by the end of 2012. Over the same period, the Russell 2000 portfolio grows to only $1,711 and the NASDAQ portfolio ends 2012 valued at a significantly lower $747. Correspondingly, the active mean–variance strategy grows to $3,530 as well as possessing less risk than the equity portfolios.

Next, we analyze the performance of the futures portfolios and the various market indexes for each

EXHIBIT 7

Returns and Standard Deviations for Stock Indexes and Portfolios at a 20% Targeted Standard Deviation

	S&P 500		Russell 2000		NASDAQ		3-Month Portfolio		6-Month Portfolio		1-Year Portfolio	
Year	**Return**	**Stdev**	**Return**	**Stdev**	**Return**	**Stdev**	**Return**	**Stdev**	**Return**	**Stdev**	**Return**	**Stdev**
1991	26.31	14.33	43.68	12.44	56.86	14.85	–2.11	16.34	12.63	10.45	–13.87	16.30
1992	4.46	9.72	16.36	10.01	15.45	13.02	1.35	9.79	–6.33	10.05	–8.87	9.69
1993	7.06	8.62	17.00	8.80	14.75	11.52	16.59	11.55	2.34	10.57	8.46	10.55
1994	–1.54	9.84	–3.18	10.01	–3.20	11.60	7.48	12.78	5.87	15.08	–6.20	14.60
1995	34.11	7.81	26.21	8.18	39.92	13.31	13.46	11.07	5.46	10.57	18.29	10.52
1996	20.26	11.83	14.76	10.72	22.71	15.46	21.29	16.51	17.88	17.13	18.32	16.65
1997	31.01	18.17	20.52	13.06	21.64	18.58	3.44	14.71	3.17	17.11	3.48	17.72
1998	26.67	20.29	–3.45	20.14	39.63	26.45	–16.35	15.10	–11.35	16.60	–28.69	13.58
1999	19.53	18.07	19.62	14.23	85.59	27.28	27.44	15.89	48.79	16.39	40.19	16.71
2000	–10.14	22.22	–4.20	29.89	–39.29	48.80	36.29	18.60	44.13	18.36	42.29	21.49
2001	–13.04	21.38	1.03	23.17	–21.05	43.21	–30.30	18.49	–27.86	18.32	–21.46	18.31
2002	–23.37	26.03	–21.58	25.14	–31.53	34.45	22.14	16.31	22.17	17.65	30.63	20.32
2003	26.38	17.07	45.37	18.89	50.01	22.26	19.80	18.13	22.20	17.50	26.41	19.13
2004	8.99	11.09	17.00	17.95	8.59	16.96	9.97	19.76	4.21	20.42	11.88	22.31
2005	3.00	10.28	3.32	16.15	1.37	12.51	22.18	19.95	25.21	21.54	15.90	20.72
2006	13.62	10.01	17.00	17.35	9.52	14.16	4.70	16.49	–0.21	19.36	3.33	20.01
2007	3.53	15.95	–2.75	20.37	9.81	17.37	38.19	16.94	34.11	17.02	35.96	16.88
2008	–38.49	41.05	–34.80	46.46	–40.54	41.14	–26.75	22.13	–37.64	29.99	–40.70	31.86
2009	23.45	27.29	25.22	36.28	43.89	28.30	17.18	16.50	26.60	14.52	26.11	16.28
2010	12.78	18.05	25.31	25.09	16.91	19.72	15.16	20.24	41.26	18.17	14.57	16.17
2011	0.00	23.27	–5.45	33.05	–1.80	25.23	–5.33	20.47	–14.82	19.72	–5.63	20.59
2012	13.41	12.72	14.63	17.00	15.91	14.95	9.98	16.34	–7.07	16.19	10.15	15.58

Notes: This exhibit reports the annualized out-of-sample results for each year for portfolios targeting at a 20% standard deviation with various rebalancing frequencies. Annual performance of three equity indexes is also reported for comparison purposes. All returns and standard deviations are in percentage terms.

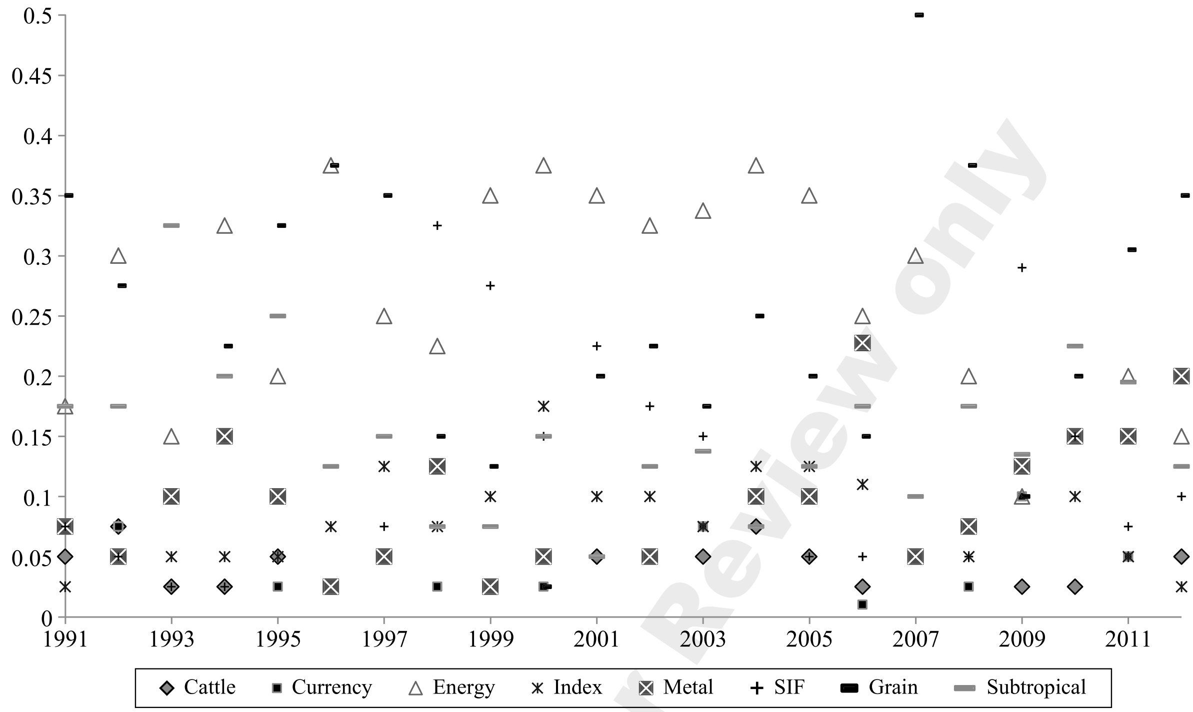
year from 1990 to 2012. Exhibit 7 presents the risk–return characteristics of the three equity indexes and of the optimized futures portfolios for a targeted 20% standard deviation.[17] In the long run, the equity indexes and portfolios behave very similarly in terms of the frequency of negative returns. In particular, both the S&P 500 and the optimized futures portfolio rebalanced quarterly show five negative return years, whereas both the NASDAQ and the futures portfolio rebalanced every six months show six negative years. Finally, both the Russell 2000 and the portfolio rebalanced annually show seven negative returns. However, in terms of weathering market downturns, the optimized portfolios perform better than the equity indexes because of less severe drawdowns. For example, although the NASDAQ performed extremely well in the 1990s, it did poorly after the new decade started, with five *large* negative returns after the year 2000. Comparatively, during the market downturns of 2000 and 2008, the three-month commodity futures portfolio significantly outperformed all equity indexes, even yielding a positive return in 2000 and a much smaller negative return in 2008. Additionally, the standard deviation of all equity indexes often changes drastically from year to year, whereas the standard deviations of the futures portfolios are more stable. Overall, these results show that the optimized commodity futures portfolios display more stability in terms of both risk and return than pure traditional equity markets.

PORTFOLIO WEIGHTS

An important question is whether the portfolio weights themselves are relatively stable over time. We determine the weight stability by examining the weight dynamics of the most volatile futures portfolio

EXHIBIT 8

Portfolio Weights in Different Groups with a Three-Month Rebalancing Period at a 20% Targeted Standard Deviation



Note: The interest rate group is removed from this graph since it is included in the portfolios in years 2001, 2008, and 2009 only.

with the highest level of rebalancing from 1990 to 2012, namely the three-month futures portfolio targeting a 20% standard deviation. Owing to the large number of individual contracts, we group the futures contracts by category and show the weights by category type. Exhibit 8 illustrates the evolution of these dollar weights over time. The energy group consistently displays a relatively large weight, followed by the grain futures category. The weights for the metal, cattle, and currency futures categories are typically small for these mean–variance portfolios, with interest rate futures only appearing occasionally. Also, note that the individual futures weights change even when category weights seem relatively stable, with changes in the weights being the basis for the superior performance of the shorter-term rebalancing results. For the mean–variance portfolios with standard deviations below 20%, the weights of the lower-risk assets (such as currency, interest rates, and metals) typically increase.[18] Overall, the results do demonstrate reasonably stable portfolio weights within the various asset groups. Exhibit 9 confirms these conclusions by providing weight statistics for the 20% standard deviation case, a sort of "worst case scenario," since more volatile portfolios tend to see a higher level of fluctuation in the weights. The results show how, while the average (mean) weights obviously vary by category, their fluctuations are fairly low as their respective standard deviations are proportionally small.

EXHIBIT 9

Statistics for Portfolio Weights in Different Groups with a Three-Month Rebalancing Period at a 20% Targeted Standard Deviation

(In Percent)	Mean	St. Dev	Min	Max
Cattle	4.22	1.76	2.50	7.50
Currency	4.60	3.15	1.00	10.00
Energy	26.49	9.22	10.00	37.50
Index	8.05	4.06	2.50	17.50
Interest	5.83	3.82	2.50	10.00
Metal	9.64	5.49	2.50	22.75
SIF	12.72	9.63	2.50	32.50
Tropical	15.19	6.38	5.00	32.50
Grain	25.25	11.16	2.50	50.00

EXTREME LOSSES

Recent market conditions raise the issue of investment tail risk, since futures returns are typically not normally distributed. Thus, extreme losses for futures can exceed what would be expected under Gaussian distributional assumptions, often described in terms of nonnormal skewness and kurtosis. Therefore, we examine the tail risk of our futures portfolios and compare the result with that of the benchmark equity indexes.

A first potential measure of tail risk is a simple nonparametric annualized estimate of the return corresponding to the 5% lower tail of the daily returns distribution for each portfolio and benchmark; however, this approach does create a scaling issue owing to converting daily extreme losses to annualized losses.[19] In order to circumvent the scaling issue, we focus on a second measure of tail risk, based on an extended four moment Modified Value-at-Risk (MVaR) calculation. Fortunately, the MVaR does not depend on any distributional assumptions,[20] with its definition provided in Equation (2):

$$\begin{aligned}\text{MVaR} = \mu_p - \bigg(& z_c + \frac{1}{6}(z_c^2 - 1)S_p + \frac{1}{24}(z_c^3 - 3z_c)K_p \\ & - \frac{1}{36}(2z_c^3 - 5z_c)S_p^2 \bigg)\sigma_p \qquad (2)\end{aligned}$$

where μ_p, σ_p, S_p, and K_p are the first four moments of portfolio P, and z_c is the number of standard deviations specifying the probability level associated with the modified VaR. When the return distribution is normal, the Modified VaR collapses to the traditional VaR. In this study, all MVaRs are calculated using a 95% confidence level with annualized returns, standard deviation, skewness, and kurtosis. For the annualization of the skewness and kurtosis, we once again implement the cumulant approach of Meucci [2010].

Exhibit 10 shows that the S&P 500 Index possesses an average annual MVaR of −26%, with the Russell 2000 and NASDAQ exhibiting an average annual MVaR of −28% and −32%, respectively. In contrast, the mean–variance futures portfolios targeting standard deviations of 5%, 10%, 15%, and 20% reveal MVaRs of −7%, −13%, −20%, and −24%, respectively. Focusing on 2008, we also observe that the MVaR returns of the commodity futures portfolios are roughly between two and ten times *smaller* in magnitude than those of the equity market benchmarks. These results confirm our previous findings from the empirical tail-risk measure, namely that the mean–variance futures portfolios are much less likely to experience potentially catastrophic losses than any of the equity benchmarks.

CONCLUSIONS

We investigate whether individual commodity futures contracts (rather than commodity indexes) in conjunction with the Markowitz optimization model provide tangible out-of-sample benefits relative to an equity portfolio, in spite of the recent increased correlations of commodity futures with financial markets. Our results support the inclusion of futures contracts both to reduce risk attributable to low correlations between contracts and to enhance returns. Moreover, we examine the effect of risk levels, rebalancing intervals, tail risk, and time periods on the results. Previous studies normally only report in-sample results and ignore the resultant (in)stability, rebalancing frequency, and tail risk of the optimal out-of-sample portfolios.

Our results for the simple correlation analysis reveal high correlation levels between futures within the same category type, but low correlations for futures contracts from different categories; thus, diversification benefits exist from including individual futures contracts belonging to different categories of assets. Examination of the out-of-sample performance of the optimized portfolios shows that including commodity futures to a portfolio often leads to superior performance compared with equity benchmarks, especially when various rebalancing frequencies, different targeted risk levels, and different sample periods are examined. The mean–variance futures portfolios are also fairly stable over time, with less variation in their standard deviation compared to equity indexes, as well as possessing reasonably stable category weights. Finally, potential extreme losses are consistently smaller for futures portfolios than for various equity index benchmarks. Consequently, including individual futures in one's portfolio can often yield better performance, higher Sharpe ratios, lower volatility levels, and a significantly reduced chance of an extreme loss. These results provide evidence on how portfolio managers, ETF providers, and investors can improve the performance of an equity portfolio. Perhaps most importantly, it shows that including nonequity futures contracts can substantially reduce risk, including extreme risk.

EXHIBIT 10

Modified VaR for Mean–Variance Futures Portfolios (three-month rebalancing frequency)

Year	S&P 500	Russell 2000	NASDAQ	5% stdev Portfolio	10% stdev Portfolio	15% stdev Portfolio	20% stdev Portfolio
1991	–4%	12%	17%	–11%	–27%	–23%	–31%
1992	–15%	–4%	–11%	–14%	–17%	–18%	–18%
1993	–10%	–1%	–8%	–8%	–12%	–10%	–7%
1994	–21%	–23%	–26%	–8%	–9%	–15%	–17%
1995	14%	8%	8%	1%	–5%	–9%	–9%
1996	–4%	–7%	–9%	–3%	–12%	–19%	–12%
1997	–7%	–6%	–15%	–8%	–18%	–29%	–25%
1998	–14%	–41%	–15%	–15%	–32%	–41%	–47%
1999	–17%	–9%	11%	–3%	–4%	–6%	–6%
2000	–53%	–60%	–136%	–9%	–12%	–10%	–4%
2001	–55%	–43%	–102%	–16%	–31%	–59%	–72%
2002	–76%	–71%	–101%	–3%	–7%	–12%	–11%
2003	–9%	2%	–1%	–1%	–5%	–14%	–16%
2004	–13%	–18%	–24%	–2%	–6%	–20%	–28%
2005	–17%	–28%	–23%	0%	–10%	–13%	–18%
2006	–7%	–18%	–18%	–5%	–13%	–22%	–27%
2007	–27%	–41%	–24%	–11%	–3%	0%	0%
2008	–122%	–124%	–126%	–12%	–29%	–50%	–72%
2009	–30%	–44%	–16%	–9%	–11%	–8%	–15%
2010	–22%	–24%	–22%	7%	3%	–10%	–24%
2011	–43%	–66%	–49%	–7%	–22%	–35%	–44%
2012	–12%	–19%	–14%	–5%	–14%	–19%	–22%
Average	–26%	–28%	–32%	–7%	–13%	–20%	–24%

Notes: This exhibit reports the annual tail risk measured by Modified VaR for portfolios rebalanced at a three-month frequency with various targeted risk levels. The same annual modified VaR is also reported for three equity indexes for comparison purposes.

ENDNOTES

[1]Although inverse-market exchange-traded funds can also display negative correlation properties, their long-term expected returns are, by definition, negative owing to inflation effects and market movements, whereas the correlations of long commodity positions—often used as an inflation hedge—are positive.

[2]The diversification benefits often were shown in terms of basic correlations, typically with commodity indexes. Initial studies include Bodie and Rosansky [1980] and Anson [1998].

[3]As with all models, the Markowitz mean–variance optimization has restrictive assumptions, in particular normally distributed returns. However, studies find that mean–variance–skewness–kurtosis (MVSK) optimization models are not superior to the traditional Markowitz mean–variance model. In fact, Bergh and Rensburg [2008] find that the MVSK efficient portfolios have lower returns than the mean–variance portfolios. Furthermore, we analyze the effect of both skewness and kurtosis in the mean–variance portfolios by analyzing its higher moment risk. Therefore, our contention is that an empirical analysis is the only approach to examine how the Markowitz model performs.

[4]Lintner [1983] finds low correlations between the performance of commodity trading advisors (CTAs) and that of a stock and bond portfolio. Kaplan and Lummer [1998] determine that the Goldman Sachs Commodity Index (GSCI) total return is negatively correlated with stocks and bonds, and Schneeweis and Georgiev [2002] show that futures accounts managed by CTAs are negatively correlated with the S&P 500 Index, especially during market downturns. More recent studies do not always find negative correlations between equity and commodity markets, although correlations are typically low. For example, You and Daigler [2013] find a low but positive correlation between equity and commodity index futures, and Tang and Xiong [2012] conclude that in the recent past, correlations between commodity and financial

contracts, as well as among different commodity futures, increased.

[5]Employing heating oil and the S&P 500, Erb and Harvey [2006] generate a higher return with a simple rebalancing of the portfolio to keep the weights of the two individual futures equal. However, this approach considers only the weights of two futures contracts. Here, we examine the effect of rebalancing using an optimization technique. Employing five individual futures contracts, Daskalaki and Skiadopoulos [2011] find no out-of-sample benefits from adding one individual commodity futures contract to the equity assets. Overall, these and similar studies that employ only a few futures contracts ignore the potential benefit resulting from low correlations between individual futures.

[6]Some futures do not start trading until after 1990; however, we still include them in our sample because of their importance. For example, the S&P Mid-Cap 400 futures began trading in 1992, the Mexican peso futures in 1995, and NASDAQ futures in 1996. Futures contracts on the DJIA did not start until 1997.

[7]We do not consider transaction costs in our article since futures are zero-cost investment vehicles, resulting in a round trip cost of less than 0.01% of the nominal value of the futures. Alternatively, the transaction costs for cash investments can be significant.

[8]After 2007, the GSCI was changed to the S&P GSCI.

[9]The in-sample optimal weights are applied at the beginning of the out-of-sample period and are allowed to evolve until the portfolio is re-optimized. The development of the weights during the out-of-sample period allows the weight of each asset to fluctuate daily based on the performance of that asset relative to that of the entire portfolio (thus no rebalancing is done within the time period); the resulting annualized out-of-sample results reported in our next section are therefore an accurate representation of the true performance that an investor would actually experience when they do not dynamically trade their investment. Since investors could employ different time intervals to restructure the weights in their portfolio, we examine different rebalancing time intervals. Although some investors might trade within such intervals, we do not examine the effect of combining optimized portfolios with trading performance. Finally, the times series of the obtained out-of-sample returns does not contain any omissions or gaps, since at every re-optimization time the out-of-sample period just ended is employed as the next in-sample period.

[10]All return distribution moments in this article are annualized from their daily counterparts. However, whereas annualizing the first two moments is straightforward (multiplying the daily first moment by 252 and the daily second moment by the square root of 252), annualizing skewness and kurtosis presents more of a challenge. Positive shocks typically offset negative ones, and as daily shocks are incorporated over time, the central limit theorem dictates that the distribution of yearly returns becomes more normal than the distribution of daily returns. Consequently, the skewness and excess kurtosis of the distribution of yearly returns will be less significant than those of the daily returns. So whereas the first two daily moments become *larger* when annualized, the third and fourth moments actually become *smaller*. Note that this is analogous to saying that the true annual Value-at-Risk will not be as severe as the annualized version of a daily one. Recognizing this fact, we use Meucci's [2010] cumulant-based approach designed to allow the conversion of higher-order moments from one frequency to another.

[11]We also compute 100-day rolling-window correlations, which are available upon request. Overall, these correlations are similar to the one-year rolling-window correlations.

[12]For example, in the case of semiannual rebalancing, six months of daily data are used to construct the portfolio, with the resulting investment portfolio left untouched for an additional six months, after which the portfolio is re-optimized.

[13]The distinction between the arithmetic and the more appropriate geometric return is particularly important during volatile times, since the higher the volatility, the higher the difference between the two. Stated differently, the higher the volatility, the more misleading the arithmetic return. For example, a loss of 40% followed by a gain of 60% yields a positive arithmetic average return of 10%. However, this scenario in fact yields an overall *loss* of 4%, which would indeed be reflected by a negative geometric average return of −2.02%.

[14]Optimization alpha refers to the higher return achieved from optimizing the portfolios of futures contracts over the return obtained from individual futures returns. Such optimized portfolios also return less risk.

[15]See http://www.cmegroup.com/trading/equity-index/report-a-cost-comparison-of-futures-and-etfs.html.

[16]We employ the 2000–2012 period to concentrate on the most recent past and not to avoid the effects of the bubble that occurred in the stock market in the latter 1990s. Moreover, the 2000–2012 period emphasizes how using a portfolio of futures contracts protects an equities portfolio from inferior performance.

[17]We focus on the futures portfolios targeting a 20% standard deviation for comparison purposes, since this is the risk level most closely matching the equity indexes.

[18]Lower volatility mean–variance portfolio results are omitted, but are available upon request.

[19]Converting daily returns to annualized returns typically yields extremely large values. To illustrate, let us assume that the daily return corresponding to the 5% lower tail is a large but reasonable loss of 2% in one day. Annualizing

this value by multiplying it by the 252 trading days in a year creates an annualized equivalent loss of 504%. The reality is that, over the course of a year, the daily positive shocks will in large part be offset the negative ones, and therefore the true annualized value-at-risk will be much less severe than the annualized value of the daily one.

[20]Favre and Galeano [2002]; Bali, Gokcan, and Liang [2007]; and Liang and Park [2007] use the Cornish–Fisher expansion to extend the Value-at-Risk (VaR) concept to a four-moment MVaR to explicitly incorporate the presence of nonnormal skewness and kurtosis.

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