Effect of intervalling and skewness on portfolio selection in developed and developing markets

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Abstract

Based on several research studies and in particular the theoretical study of Prakash, de Boyrie, Hamid and Smyser (1997), it is known that the variance as well as the skewness of the probability distribution of rates of return *increases* if the investors' investment interval increases. In the present study, using the portfolio selection procedure developed by Lai (1991) under the presence of skewness and subsequently used by Chunhachinda, Dandapani, Hamid and Prakash (1997) and Prakash, Chang and Pactwa (2003), we find that the selection of investment interval (e.g. daily, weekly vs. monthly) significantly changes not only the optimal allocation of weights, but also the number of markets selected in the portfolio.

JEL Classification: G11, G15

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1. Introduction

The present study, in large part, is inspired by the abundance of studies that have appeared lately on skewness and intervalling in the literature. For example Harris, Kucukozmen, and Yilmaz (2004) show that a recently introduced probability distribution known as "skewed generalized t (SGT) distribution, a distribution that allows wide range of skewness and kurtosis….. offers a substantial improvement in the fit of both GARCH and EGARCH models"[1]. Sengupta (2003), using a variety of efficiency tests, also finds that the presence of skewness in the rates of return of mutual funds based on new technology significantly affects their performance. Furthermore, Parhizgari, Dandapani and Prakash (1993) and Josey, Brooks and Faff (2001) offer empirical evidence that the choice of intervalling in datasets (e.g., daily versus weekly versus monthly, etc.) does affect the various return generating (such as linear and quadratic) and asset pricing (such as arbitrage pricing theory) models. Even though there exist several studies on portfolio choice under skewness (see for example, Chunhachinda, Dandapani, Hamid and Prakash, 1997 and Prakash, Chang and Pactwa, 2003), to the best of our knowledge no study specifically addresses the effect of intervalling as well as the effect of skewness on the portfolio choice problem simultaneously, along with a large worldwide data set. In view of this, we reexamine the effect of intervalling in portfolio selection in the presence of preference for positive skewness by the investor. Although we employ the same procedure for portfolio selection under skewness preference as in Lai (1991), Chunhachinda, Dandapani, Hamid and Prakash (1997), and Prakash, Chang and Pactwa (2003), this paper defers markedly from the above cited studies in the sense

that we exclusively address the question of the effect of investment interval selection

on the allocation of optimal weights to individual assets in a portfolio recognizing

the preference for positive skewness. Following Parhizgari, Dandapani and Prakash

(1993), we use the phrases "investment horizon problem", "holding period problem"

and "intervalling effect" interchangeably. As Parhizgari et al. (1993) note:

> "The finance literature is relatively rich in its coverage of the effect of the investment horizon on performance measure. This effect has been named alternatively: the intervalling effect, the investment horizon problem, and the holding period problem. Stated simply, the problem is that estimates of the various measures of beta and the performance indices using a *given* data set could be 'interval' independent. The interval is basically the time mode of the data set, e.g. daily, weekly, monthly, quarterly, annually, etc. Several researchers, most notably Levhari and Levi (1977), Ang and Chua (1979), and Levy (1981) have shown that in testing the performance measures of the security or portfolio using CAPM, the arbitrarily chosen investment horizon affects the resultant estimates. The performance index as measured by the reward to variability ratio increases with the investment horizon for all stocks and in the market model the estimate of systematic risk increases with the investment horizon for aggressive stocks and decreases for defensive stocks. Hence it is theorized that most of the empirical studies evaluating performance contain a mathematical bias in measuring systematic risk. This bias is attributed to the choice of the investment horizon."

It is evident from the above quote and references cited therein that the

various statistical measures that form the backbone of the financial decision making

process are affected by the choice of investment interval, such as daily versus weekly

versus monthly, etc[2]. Many research studies have appeared in the literature that

discuss the effect of intervalling on various statistical measures, such as beta,

variance, skewness, Sharpe's performance index, etc. (see for example Smith, 1978;

Hawawini, 1980a, 1980b; Levy, 1972; Lee and Leuthold, 1983; Handa, Kothari and

Wasley, 1993; Parhizgari, Dandapani and Prakash, 1993; Martinkainen, Pertunnen,

Yli-Olli and Gunasekaren, 1994; etc). However, we are aware of only Prakash, de

Boyrie, Hamid, and Smyser (1997, PDHS hereafter) who provide a purely theoretical

discourse on the effect of intervalling on variance and skewness. Furthermore, the

list of seminal research studies that have appeared in the literature using variance as

well as preference for positive skewness in the portfolio allocation are Meric and

Meric (1989), Lai (1991), Chunhachinda et al. (1997) and Prakash et al. (2003).

However, to the best of our knowledge, no research has appeared in the literature

that deals with the intervalling effect and utilizes variance combined with preference for positive skewness in the rates of return. The motivation for this research comes from the fact that if the choice of investment interval affects the variance and skewness, it then seems intuitive that it will also affect the allocation of optimal weights for the assets in the portfolio. In this paper, we make an attempt to integrate the intervalling effect on the portfolio selection problem. Note that we refrain from reviewing the literature and the empirical methodology in detail since it can be found in Chunahachinda et al. (1997) and Prakash et al. (2003).

Using market index data from 37 developed and developing countries, we apply the goal programming techniques of Lai (1991) to obtain the optimal portfolio allocation for daily, weekly and monthly investment intervals. We find that both the allocation of weights and the selection of markets significantly affect the optimal portfolios' composition.

To maintain continuity, the theoretical development of the intervalling effect in PDHS (1997) is briefly discussed in section 2. Section 3 describes the data used in our empirical analysis. The empirical verification of the intervalling effect on variance and skewness of returns is conducted in Section 4. In section 5, we present a brief description of the multi-objective goal programming model, originally developed by Lai (1991), and go on to describe the empirical results obtained using the multi-object goal programming. The paper concludes with some remarks in section 6.

2. The intervalling effect on the variance and skewness of return distributions[3]

PDHS (1997) define the one period rate of return during the interval $(j\text{-}1)$ to j as

$$\tilde{r}_j = \frac{\tilde{P}_j - P_{j-1}}{P_{j-1}} \qquad (1)$$

where $\tilde{P}_j$ (a random variable) is the expected price to prevail in time period j, and P_{j-1} (a non-random number) the price in period $j\text{-}1$. The wealth ratio is then given by

$$\tilde{R}_j = 1 + \tilde{r}_j \qquad (2)$$

PDHS (1997) examine the theoretical distribution of $\tilde{R}_j$ under the stochastic proportionate effect jump process where the jump process is defined thus

"Let the initial value of the price be P_0 and through a jump process a value P_j is attained at the jth jump. Let P_T be the final value at the Tth jump where the process terminates[4]. Assuming that at the jth step $(j=1,2,\ldots\ldots,T)$ the random change in the variable is a random proportion of the most immediate attained value, i.e.

$$\tilde{P}_j - P_{j-1} = \tilde{r}_j \phi(P_{j-1}) \qquad (3)$$

where $\tilde{r}_j$'s are mutually independent for all j except when $\phi(P_{j-1}) = 1$. In this paper our interest is in the distribution of (1) which is a special case of (3) when $\phi(P_{j-1}) = P_{j-1}$. Imposing the condition $\phi(P_{j-1}) = P_{j-1}$, the process defined by (3) becomes

$$\tilde{P}_j - P_{j-1} = \tilde{r}_j (P_{j-1}) \qquad (4)$$

which obeys the law of proportionate effect[5].

Equation (4) can be expressed as

$$\tilde{P}_j - (1 + \tilde{r}_j)P_{j-1} = 0 \qquad (5)$$

Putting $j=1,2,\ldots\ldots,T$ and solving recursively we get

$$\tilde{P}_T = P_0(1 + \tilde{r}_1)(1 + \tilde{r}_2)\ldots\ldots(1 + \tilde{r}_T) \qquad (6)$$

or $\qquad \ln\dfrac{\tilde{P}_T}{P_0} = \displaystyle\sum_{j=1}^{T} \ln(1 + \tilde{r}_j)$

and $\qquad \ln \tilde{R}_0^T = \displaystyle\sum_{j=1}^{T} \ln \tilde{R}_j \qquad (7)$

where $\tilde{R}_0^T$ denotes the holding period (from 0 to T) wealth ratio."

PDHS (1997) then obtain the probability distribution of $\ln \tilde{R}_0^T$. Note that the difference between superscript and subscript defines the length of the interval. Using the Gnedenko (1962) form of the Law of Large Numbers they show that, since

$Y = \ln \tilde{R}_0^T = \sum_{j=1}^{T} \ln \tilde{R}_j$ is asymptotically normally distributed. Since

$Y = \ln \tilde{R}_0^T = \ln(\prod_{j=1}^{T} \tilde{R}_j)$ is normal then $\prod_{j=1}^{T} \tilde{R}_j$ will be lognormal with parameters

$\theta = \sum_{j=1}^{T} \mu_j$ and $\xi^2 = \sum_{j-1}^{T} \sigma_j^2$, where μ_j, and σ_j are the mean and standard deviation of

r_j, respectively, with the following values for mean, variance and skewness of the lognormal distribution:

Mean: $M = \exp(\theta + \xi^2/2)$

Variance: $V = \exp(2\theta + \xi^2)(\exp \xi^2 - 1)$

Skewness: $SK = (\exp \xi^2 - 1)^{3/2} + 3(\exp \xi^2 - 1)^{1/2}$

After deriving the above statistical measures they argue that

" ……the skewness and the variance of the distribution depends on the parameter ξ^2 and since $\xi^2 > 0$ and $(\exp \xi^2 - 1) > 0$, the larger the value of ξ^2, the larger will be the variance and skewness.

Since $\xi^2 = \sum_{j=1} \sigma_j^2$, it implies that as T (the investment interval) increases, the value of parameter ξ^2

associated with $\prod_{j=1}^{T} \tilde{R}_j$ also increases. Thus from the variance of the distribution of $\prod_{j=1}^{T} \tilde{R}_j$, it is

expected that $V(\tilde{R}_j) \leq V(\prod_{j=1}^{5} \tilde{R}_j) \leq V(\prod_{j=1}^{10} \tilde{R}_j) \leq V(\prod_{j=1}^{20} \tilde{R}_j) \leq V(\prod_{j=1}^{40} \tilde{R}_j)$, or in holding period wealth notation of expression (7) the above inequality can be written as

$$V(\tilde{R}_0^1) \leq V(\tilde{R}_0^5) \leq V(\tilde{R}_0^{10}) \leq V(\tilde{R}_0^{20}) \leq V(\tilde{R}_0^{40}) \qquad (9)$$

where the difference between the superscript and the subscript denotes the holding period.
Similarly the same inequality as obtained for variance will be maintained for skewness expressed as:

$$SK(\tilde{R}_0^1) \leq SK(\tilde{R}_0^5) \leq SK(\tilde{R}_0^{10}) \leq SK(\tilde{R}_0^{20}) \leq SK(\tilde{R}_0^{40}) \text{ "} \qquad (10)$$

From the above discussion it is obvious that the choice of interval will affect the values of the variance and skewness.

3. The data

We collect daily, weekly, and monthly data on international indices from Datastream from July 1st 1993 to May 30th, 2005, from 37 countries spanning over the five continents. The price series for each country index are subsequently converted to return series. The countries included in this study are:

Developed countries:

Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Norway, Spain, Sweden, Switzerland, UK, Australia, Canada, New Zealand, US, Japan, Singapore, and Hong Kong.

Developing countries:

China, Indonesia, Korea, Malaysia, Philippines, Taiwan, Thailand, Argentina, Brazil, Chile, Mexico, Venezuela, Portugal, Turkey, and Poland.

Thus there are 22 developed and 15 developing countries in this study. Summary statistics of our data sample can be found in Table 1.

4. The empirical verification of the intervalling effect on variance and skewness

According to expressions (9) and (10), the variance and skewness for the returns should increase as the investment interval increases. In fact this is exactly, what Fisher and Lorie (1970) reported a long time ago (see their Table 3 pages 106 and 107, column 17). It is clear from their table that inequality (9) is satisfied for all intervals considered except in one case. Furthermore, inequality (10) is satisfied in almost all cases except for one ten-year period over five-year period. Thus, Fisher and Lorie's finding confirm the Prakash, de Boyrie, Hamid and Smyser (1997)

theoretical derivation. Furthermore, Chunhachinda et al. (1997) Table 1 shows that all the variances of weekly rates of return for 14 international capital markets over a period of January 1988 to December 1993 are less than their monthly counterparts. Similarly, the weekly skewness is less than their monthly counterparts in all cases except one[6]. Prakash et al. (2003) also report the standard deviations and skewnesses for (nominally) annualized weekly and monthly returns. At first glance, the standard deviations seem to be just the opposite of the theoretically predicted values. However, if we convert the annualized weekly and annualized monthly standard deviations to the standard deviations of actual interval returns, their findings are completely consistent with the findings of Fisher and Lorie (1970), and Chunhachinda et al. (1997)[7]. However, unlike Fisher and Lorie (1970) and Chunhachinda et al. (1997), this study provides no clear cut evidence whether skewness increases with an increase in the investment interval.[8].

Our empirical results are consistent with the findings of Prakash et al. (2003), as can be seen in Table 1.

[Insert Table 1 here]

It is clear from Table 1 that for periodic returns the computed values of daily variances for all 37 capital markets of the world are less than their weekly monthly counterparts and in turn less than their monthly counterparts. However, the evidence is not as conclusive for the skewness. The conflicting findings of Prakash et al. (2003) and the present study are not surprising. The datasets used in the present and Prakash et al. (2003) studies are overlapping. Prakash et al. (2003) used data from July 1993 through December 2000 whereas we use data from July 1993 to May 2005. The data used by Fisher and Lorie were for pre-1970 periods whereas Chunhachinda et al. (1997) used data from January 1988 to December 1993. At the

present time, we do not have a good explanation for the discrepancies observed in these findings.

5. Solving the multi-objective portfolio problem and empirical results

Our multi-objective portfolio problem is the same as in Prakash, Chang and Pactwa (2003). That is, we essentially solve the multi-objective (polynomial) goal programming problem:

$$\min_{w}\{k_p(w_{MV}) - k_p(w)\}^a + \{s_p(w_{MV}) - s_p(w)\}^b$$

$$\text{s.t. } w \geq 0, \ w'\mathbf{1} = 1 \text{ and } v_p(w) = 1$$

where $\mathbf{1}$ is the unit vector, an (n x 1) vector;

w is the portfolio weights, an (n x 1) vector;

$k_p(w)$ is the expected excess return on portfolio;

$v_p(w)$ is the variance of portfolio;

$s_p(w)$ is the third central moment of distribution of portfolio returns;

w_{MV} solves $\max_w k_p(w)$ s.t. $w \geq 0$ and $v_p(w) = 1$, the solution to the mean–variance efficient portfolio; and

w_{SV} solves $\max_w s_p(w)$, s.t., $w \geq 0$ and $v_p(w) = 1$, the solution to the skewness–variance efficient portfolio;

$w_{PGP}(a,b)$ is the solution to polynomial goal programming problem, where a and b are parameters in the objective function.

Selection of integer values for a and b will determine the portfolio selection choice. For example, if we select $a = 1$ and $b = 0$, it will represent the mean-variance

efficient portfolio. Similarly choosing $a = 1$ and $b = 1$ will depict the selection of the mean-variance-skewness portfolio.

Using the procedure described above, we obtain the optimal portfolio allocation for the 22 developed and 15 developing countries' market indices, a total of 37 markets.

5.1 Portfolio allocation among developed markets

In Table 2, we provide the optimal portfolio allocation among the developed markets for the rates of return. Furthermore, the table also contains the weight allocation for daily, weekly, as well as monthly returns for different values of parameters a and b[9]. Even though all the tables provide the allocation for various values of parameters a and b, our main concern is when $a = 1$, $b = 0$ (mean-variance portfolio allocation) and when $a = 1$, $b = 1$ (mean-variance-skewness portfolio allocation)[10]. In Table 2, we provide the portfolio allocation for daily (panel 1), weekly (panel 2) and monthly (panel 3) returns. Under the mean-variance framework ($a = 1$, $b = 0$) in developed markets, the optimal portfolio allocations for daily returns are fairly evenly distributed among all 22 developed markets in the sample, with the highest allocation to Finland (8.31%) and the lowest allocation to US (2.60%). In the case of weekly returns, the portfolio allocations are spread over 16 countries with Finland (12.61%) receiving the highest allocation followed by Demark (12.07%). But in the monthly returns case, the allocation happens only in 9 out of 22 developed markets with Denmark receiving the highest weight (35.90%), followed by US (24.05%). Thus, it is obvious that the selection of investment interval does have a large impact on the allocation weights among the different markets.

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In the mean-variance-skewness preference ($a = 1$, $b = 1$) case, the number of markets in which it is optimal to invest is not even the same as the number of markets in the mean-variance portfolio allocation problem (for weekly returns, five, and for monthly returns, three). The countries that are selected for weekly returns are Belgium (4.47%), Finland (5.66%), Italy (47.87%), Switzerland (25.96%) and Hong Kong (16.05%), whereas the allocation weights for monthly returns are Austria (2.87%), Switzerland (40.97%) and Hong Kong (56.15%). Thus, it is obvious that the choice of investment interval drastically changes not only the optimal weights but the markets as well. For example, Switzerland and Hong Kong appear in both the monthly and the weekly returns cases, albeit with drastically different allocations (25.96% and 16.05% versus 40.97% and 56.15%, respectively).

5.2 Portfolio allocation among developing markets

The weight allocations for assets in the developing markets follow essentially the same pattern witnessed earlier. For example, for pure mean-variance preference ($a = 1$, $b = 0$), the allocation in the daily return case covers all 15 markets in the sample, with the highest percentage going to Indonesia (10.10%) and the lowest to Portugal. However, in the weekly returns case the allocation is spread among eight (namely, China, Korea, Brazil, Chile, Mexico, Portugal, Turkey and Poland) out of 15 countries while the allocation in the monthly returns case is distributed among seven countries. Furthermore, unlike the case of developing countries where weekly returns yielded different countries than monthly returns did, no such pattern is observed in the mean-variance framework. Chile is the only country that fails to receive any allocation when using monthly returns, having received a mere 0.53%

allocation when using weekly returns. In seems that the increase in investment horizon reduces the number of markets included in the optimal portfolio under the mean-variance framework.

[Insert table 3 here]

In the mean-variance-skewness framework ($a = 1$, $b = 1$), the optimal portfolio in the daily return case comprises only one market, that is, Philippines receives 100% of the funds allocation. However, when weekly returns are used, the funds are allocated to seven countries (China, 80.51%, Malaysia, 0.97%, Brazil, 2.89%, Chile, 3.21%, Mexico, 3.34%, Portugal, 7.92% and Poland, 1.15%) whereas for monthly returns the mean-variance-skewness portfolio selected consists of China (90.29%), Philippines (1.70%), Thailand (2.78%) and Brazil (5.23%). Thus, unlike the pure mean-variance setting, the countries selected when using daily returns are vastly different from the ones selected when using weekly or monthly returns. This finding is contrary to the findings for mean-variance preference portfolio where many countries were selected for all three rates of returns.

5.3 Portfolio allocation among developed and developing markets

The portfolio allocation for assets in all 37 developed and developing markets combined is presented in Table 4.

[Insert table 4 here]

For pure mean-variance preference ($a = 1$, $b = 0$), once again the optimal portfolio in the daily returns case covers all the markets in the sample, with the highest percentage allocated to Turkey (4.62%), followed by Finland (4.60%) and the lowest percentage allocated to China (0.89%). On the other hand, the allocation in the weekly returns case covers 13 countries (Australia, Austria, Belgium, Canada,

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China, Denmark, Finland, Greece, Ireland, New Zealand, Spain, Turkey and US), with the highest weights going to Denmark (18.62%), Ireland (16.10%) and Canada (13.23%). In the case of monthly returns, the allocation of assets is spread across nine countries (Australia, Austria, Canada, China, Denmark, Greece, Ireland, Turkey and US), a subset of the countries selected when using weekly returns, with the highest being Denmark (35.82%), US (22.89%) and Ireland (12.94%). Note that the increase in investment interval reduces the number of markets invested in as well as changes the compositions of the optimal portfolio.

In the mean-variance-skewness framework ($a = 1$, $b = 1$), the funds are allocated to nine markets (Argentina, Hong Kong, Indonesia, Philippines, Spain, Thailand, UK, US and Venezuela) in case of daily returns, with the majority of funds going to Thailand (63.60%). However, the optimal portfolio consists of ten countries when using weekly returns (Brazil, Chile, China, Greece, Malaysia, Mexico, New Zealand, Poland, Portugal and UK), with three fourths of the funds allocated to China (76.92%); whereas for monthly returns the optimal mean-variance-skewness portfolio includes seven countries, namely Austria, Belgium, Brazil, China, Ireland, Thailand and United Kingdom, with the concentration of assets in China (77.92%) again. Note that there are only two countries, China and UK, selected in all three cases (daily, weekly and monthly). Also, when daily and weekly returns are used, more developing countries (five out of nine in daily returns case and seven out of ten in weekly return case) are selected than developed countries.

6. Conclusion

In this paper, we both empirically examine the intervalling effect on the variance and skewness of the distribution of returns and study the implication of this effect on portfolio construction. The theory predicts that the variance as well as the skewness should increase if we increase the investment interval. In other words, the variance as well as the skewness of the weekly rates of return will be smaller than that of, say, monthly rates of returns. Earlier research findings (Fisher and Lorie, 1970, Chunhachinda et al., 1997) empirically support the theoretical predictions. Our study supports the Prakash et al. (2003) empirical findings in that the empirical evidence for theoretically predicted behavior of variance pertaining to investment interval is confirmed, but the empirical evidence for skewness is mixed. Like previous studies by Chunhachinda et al. (1997) and Prakash et al. (2003), we then implement the polynomial goal programming technique to identify the optimal portfolio allocation among developing and developed markets. Some resulting optimal portfolios appear at first to lack common sense, and these are portfolio with: 1) the highest weights in very small capital markets such as Finland and Denmark, 2) weights of 40-50% or higher in a single capital market in Hong Kong, in Switzerland and in Italy, etc., and 3) finally, a seemingly implausible allocation of over 80% in China using weekly returns, and over 90% using monthly returns[11]. However, in the mean-variance-skewness setting these outcomes are actually sensible. For example, an examination of Table 1 shows China has a positive skewness, with favorable mean return and variance to match, that is 5 times the skewness calculated for the nearest country. It is thus natural for the bulk of the allocation to go to China. Overall we find that the choice of investment interval changes not only the allocation weights, but the number of markets or assets in the portfolio as well. This phenomenon is observed for both developing and developed countries.

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TABLE 1 Summary statistics – daily, weekly and monthly return

	mean return			variance			skewness			kurtosis			W-statistic			Prob<W		
	Daily	Weekly	Monthly	Daily	Weekly	Monthly	Daily	Weekly	Monthly	Daily	Weekly	Monthly	Daily	Weekly	Monthly	Daily	Weekly	Monthly
ARGENTINA	0.0002	0.0005	0.0024	0.0004	0.0021	0.0100	-0.8495	-0.2370	-0.2307	22.8456	6.4896	3.5221	13.7882	6.3390	-0.3356	0.00000	0.00000	0.36857
AUSTRALIA	0.0004	0.0019	0.0085	0.0001	0.0005	0.0026	-0.0883	-0.1574	-0.1981	6.3017	4.0250	3.0163	9.6489	2.7475	0.4081	0.00000	0.00300	0.34161
AUSTRIA	0.0004	0.0019	0.0083	0.0001	0.0004	0.0021	-0.2432	-0.1968	-0.0826	4.5185	3.1989	2.9181	8.1500	1.4716	0.0561	0.00000	0.07056	0.47761
BELGIUM	0.0004	0.0020	0.0088	0.0001	0.0005	0.0020	0.1810	-0.0508	-0.6251	6.7387	5.3161	3.8808	10.7774	5.5755	2.4034	0.00000	0.00000	0.00812
BRAZIL	0.0004	0.0022	0.0108	0.0004	0.0023	0.0131	0.0731	-0.3584	-0.0869	9.2307	4.4557	4.4773	11.6898	4.7448	2.2738	0.00000	0.00000	0.01149
CANADA	0.0004	0.0023	0.0101	0.0001	0.0006	0.0026	-0.6190	-0.4800	-0.8191	7.9072	5.6051	4.9721	11.4400	6.3816	3.1047	0.00000	0.00000	0.00095
CHILE	0.0003	0.0010	0.0048	0.0001	0.0007	0.0040	0.0403	-0.1310	-0.1643	6.4451	4.7526	4.2652	10.2277	4.5981	1.5272	0.00000	0.00000	0.06335
CHINA	0.0003	0.0035	0.0188	0.0005	0.0038	0.0259	1.7677	8.5074	6.6416	55.7048	132.8300	61.5890	15.5637	12.3190	8.8364	0.00000	0.00000	0.00000
DENMARK	0.0005	0.0027	0.0113	0.0001	0.0005	0.0023	-0.2031	-0.5594	-0.5102	5.5404	5.5681	3.8194	9.6764	5.6663	1.6286	0.00000	0.00000	0.05170
FINLAND	0.0009	0.0043	0.0185	0.0004	0.0020	0.0088	-0.2485	-0.2707	0.2986	9.4082	5.6548	4.1197	11.8939	6.1520	1.8624	0.00000	0.00000	0.03127
FRANCE	0.0004	0.0019	0.0083	0.0001	0.0006	0.0025	-0.0881	-0.0306	-0.1619	5.1572	4.5032	2.8499	9.5143	4.3028	-0.0271	0.00000	0.00001	0.48918
GERMANY	0.0003	0.0016	0.0070	0.0001	0.0007	0.0029	-0.1604	-0.1443	-0.3773	5.2235	5.1820	4.0218	9.5227	5.6163	1.8064	0.00000	0.00000	0.03543
GREECE	0.0005	0.0028	0.0122	0.0003	0.0015	0.0068	0.0989	0.3281	0.3207	6.5628	5.2990	4.2020	11.3140	6.2304	1.6119	0.00000	0.00000	0.05349
HONG KONG	0.0003	0.0017	0.0082	0.0003	0.0013	0.0071	0.2877	-0.1337	0.4240	13.1525	5.4948	5.0792	12.8501	5.4997	3.3389	0.00000	0.00000	0.00042
INDONESIA	0.0002	0.0012	0.0037	0.0008	0.0051	0.0194	0.0145	0.8958	0.4586	24.5569	18.3960	4.8372	15.2183	10.1220	2.3992	0.00000	0.00000	0.00822
IRELAND	0.0005	0.0027	0.0115	0.0001	0.0006	0.0026	-0.2592	-0.4133	-0.4827	6.7421	5.9723	3.8617	10.8857	6.3679	2.1881	0.00000	0.00000	0.01433
ITALY	0.0004	0.0020	0.0088	0.0002	0.0009	0.0041	-0.0054	0.3440	0.4925	5.3270	7.2748	3.1461	9.4419	6.7120	1.5310	0.00000	0.00000	0.06288
JAPAN	0.0000	0.0000	-0.0002	0.0002	0.0009	0.0036	0.2687	0.3756	0.3115	6.8186	3.9626	2.7833	10.1547	4.0158	1.2379	0.00000	0.00003	0.10787
KOREA	0.0005	0.0021	0.0098	0.0006	0.0035	0.0175	0.8728	-0.0621	1.2806	18.2923	10.2720	8.3359	13.7138	8.0972	4.7401	0.00000	0.00000	0.00000
MALAYSIA	0.0001	0.0004	0.0016	0.0004	0.0022	0.0107	0.2374	0.4916	0.7779	49.7505	20.2520	7.3309	15.8884	10.5470	5.0602	0.00000	0.00000	0.00000
MEXICO	0.0004	0.0016	0.0069	0.0003	0.0017	0.0081	-0.3461	-0.2948	-1.0738	16.7474	6.9221	5.6056	13.7637	6.6420	4.1555	0.00000	0.00000	0.00002
NETHERLANDS	0.0004	0.0018	0.0078	0.0001	0.0006	0.0022	-0.1319	-0.2432	-0.7903	6.6094	6.5516	3.8267	11.1058	6.9847	3.2127	0.00000	0.00000	0.00066
NEW ZEALAND	0.0003	0.0016	0.0073	0.0001	0.0006	0.0034	-0.5126	-0.2820	-0.3495	12.4075	4.7582	3.4381	11.9652	4.9176	0.9305	0.00000	0.00000	0.17606
NORWAY	0.0005	0.0025	0.0108	0.0001	0.0008	0.0036	-0.4337	-0.2637	-0.6402	6.8455	6.0065	5.0158	10.8882	6.4547	2.6739	0.00000	0.00000	0.00375
PHILIPPINES	0.0000	-0.0006	-0.0024	0.0003	0.0016	0.0091	1.3790	-0.1784	0.9192	22.5492	8.5182	7.8794	13.8211	7.7885	4.2684	0.00000	0.00000	0.00001
POLAND	0.0001	0.0020	0.0081	0.0004	0.0022	0.0107	0.1224	0.0155	0.2108	9.1762	4.4498	4.6287	11.8560	4.7867	1.9877	0.00000	0.00000	0.02343
PORTUGAL	0.0004	0.0016	0.0072	0.0001	0.0006	0.0031	-0.1407	0.0802	0.0267	6.0824	5.8383	3.1133	10.1722	5.4000	-0.2893	0.00000	0.00000	0.38616
SINGAPORE	0.0002	0.0008	0.0041	0.0002	0.0010	0.0050	0.2284	-0.1766	0.1701	10.2131	8.2715	4.9293	12.2886	7.5084	3.8429	0.00000	0.00000	0.00006
SPAIN	0.0005	0.0026	0.0113	0.0001	0.0007	0.0032	-0.1147	0.0115	0.0421	5.1317	4.4837	4.0736	9.3143	4.1353	0.7031	0.00000	0.00002	0.24099
SWEDEN	0.0006	0.0029	0.0125	0.0002	0.0012	0.0052	0.0130	0.1959	-0.2448	6.5652	7.5598	3.8298	10.6922	6.5647	0.3656	0.00000	0.00000	0.35732
SWITZERLAND	0.0004	0.0022	0.0096	0.0001	0.0005	0.0022	-0.0897	0.0762	-0.3901	6.1141	5.6997	3.9971	10.1254	5.7001	1.1050	0.00000	0.00000	0.13459
TAIWAN	0.0003	0.0009	0.0036	0.0003	0.0017	0.0084	0.0858	0.3470	0.4209	5.6234	5.4609	3.6571	10.3707	5.7619	1.5854	0.00000	0.00000	0.05644
THAILAND	0.0001	-0.0001	0.0003	0.0005	0.0029	0.0153	0.7642	0.4628	0.4579	10.2383	5.8306	4.3881	12.8011	6.7210	3.1589	0.00000	0.00000	0.00079
TURKEY	0.0009	0.0051	0.0229	0.0012	0.0061	0.0321	0.2043	0.1560	0.8475	8.3426	9.9015	5.4604	11.8756	7.8234	3.7959	0.00000	0.00000	0.00007
UK	0.0003	0.0015	0.0065	0.0001	0.0004	0.0015	-0.1358	-0.1023	-0.4009	5.3113	5.0337	2.8094	9.5009	4.9248	1.6920	0.00000	0.00000	0.04532
US	0.0004	0.0019	0.0083	0.0001	0.0005	0.0019	-0.0325	-0.4556	-0.6155	6.8570	6.6897	3.4874	10.9895	6.5350	2.4264	0.00000	0.00000	0.00763
VENEZUELA	0.0000	-0.0001	0.0016	0.0006	0.0031	0.0167	-3.1531	-1.2373	-0.2730	89.4414	19.6620	5.3430	15.6911	9.6740	3.7295	0.00000	0.00000	0.00010

TABLE 2: Polynomial goal programming: developed markets

	Daily Returns						Weekly Returns						Monthly Returns					
Parameters																		
a	1	0	1	2	1	2	1	0	1	2	1	2	1	0	1	2	1	2
b	0	1	1	1	2	2	0	1	1	1	2	2	0	1	1	1	2	2
Optimal Portfolio Composition																		
AUSTRIA	3.04%						8.77%						6.90%		2.87%		17.94%	16.74%
BELGIUM	4.22%			3.39%		5.77%	5.29%	2.74%	4.47%	3.41%	10.69%	6.42%	3.71%				0.85%	
DENMARK	3.92%						12.07%						35.90%					
FINLAND	8.31%	4.74%	7.69%	4.39%	12.46%		12.61%	4.24%	5.66%	4.42%	11.98%	8.34%	6.00%					
FRANCE	5.65%						0.09%											
GERMANY	5.70%																	
GREECE	4.50%						7.99%				1.81%		0.03%					
IRELAND	3.96%						11.26%				2.66%		17.42%					
ITALY	5.29%						0.93%	51.96%	47.87%	51.53%	27.34%	34.81%						
NETHERLANDS	5.54%																	
NORWAY	4.87%						5.92%											
SPAIN	5.35%						5.97%											
SWEDEN	6.87%	27.70%	25.43%	25.63%	22.61%	22.44%	1.76%				9.42%	18.07%						
SWITZERLAND	4.57%						6.04%	24.46%	25.96%	23.99%	20.93%	15.29%		38.32%	40.97%	39.79%	41.54%	36.89%
UK	4.30%																	
AUSTRALIA	3.28%						5.51%						2.91%					
CANADA	3.20%						7.05%						3.09%					
NEW ZEALAND	2.83%		2.42%	4.88%	4.20%		3.76%											
US	2.60%	29.83%	28.23%	26.99%	30.72%	28.51%	4.97%						24.05%					
JAPAN	3.38%																	
SINGAPORE	3.66%	33.70%	34.49%	30.73%	30.01%	43.27%						2.23%						
HONG KONG	4.96%	4.03%	1.74%	3.99%				16.60%	16.05%	16.65%	15.18%	14.84%		61.68%	56.15%	60.21%	39.67%	46.36%
Optimal portfolio statistics (all are unit variance)																		
mean	5.19%	2.77%	2.89%	2.81%	2.95%	2.92%	13.37%	8.97%	9.27%	9.00%	10.53%	9.69%	28.86%	14.37%	15.21%	14.59%	17.95%	16.79%
skewness	-0.24	0.20	0.20	0.20	0.16	0.18	-0.20	0.47	0.47	0.47	0.41	0.44	-0.50	0.51	0.51	0.51	0.42	0.47

Note: The weight in the goal programming model on deviation from maximum return is *a*, the weight on deviation from maximum skewness is *b*.

TABLE 3: Polynomial goal programming: developing markets

	Daily Returns						Weekly Returns						Monthly Returns					
Parameters																		
a	1	0	1	2	1	2	1	0	1	2	1	2	1	0	1	2	1	2
b	0	1	1	1	2	2	0	1	1	1	2	2	0	1	1	1	2	2
Optimal Portfolio Composition																		
CHINA	3.51%				1.90%	0.21%	22.11%	80.96%	80.51%	81.56%	23.96%	21.85%	21.22%	90.36%	90.29%	90.38%	83.62%	88.94%
INDONESIA	10.10%											5.11%						
KOREA	9.40%						4.99%				7.33%	3.58%	9.05%					
MALAYSIA	6.56%							1.02%	0.97%	1.06%		7.51%						
PHILIPPINES	5.44%	100.00%	100.00%	100.00%	91.23%	99.50%								2.23%	1.70%	2.07%		0.69%
TAIWAN	5.28%										0.54%	3.55%						
THAILAND	8.96%													2.73%	2.78%	2.77%	0.65%	3.06%
ARGENTINA	6.55%					0.14%												
BRAZIL	7.08%						5.20%	2.87%	2.89%	3.41%	7.73%	6.98%	4.43%	4.67%	5.23%	4.78%	7.95%	5.11%
CHILE	3.86%				4.25%	0.09%	0.53%	3.23%	3.21%	3.40%	3.25%	2.86%						
MEXICO	6.53%				2.10%		1.05%	3.30%	3.34%	3.12%	4.88%	9.41%	2.74%					
VENEZUELA	6.38%											4.81%						
PORTUGAL	2.66%				0.48%	0.05%	48.27%	7.50%	7.92%	6.18%	21.36%	9.73%	45.29%				5.16%	0.21%
TURKEY	10.95%				0.04%	0.01%	16.16%	0.00%	0.00%	0.00%	22.21%	16.62%	16.55%				2.62%	1.98%
POLAND	6.73%						1.69%	1.11%	1.15%	1.27%	8.75%	7.99%	0.72%					
Optimal portfolio statistics (all are unit variance)																		
mean	3.03%	2.60%	2.60%	2.60%	2.89%	2.65%	10.14%	6.20%	6.22%	6.18%	9.76%	8.89%	19.43%	11.87%	11.91%	11.88%	12.78%	12.24%
skewness	-0.25	1.38	1.38	1.38	1.31	1.35	1.47	8.60	8.60	8.60	1.12	1.12	1.39	6.65	6.65	6.65	6.60	6.64

Note: The weight in the goal programming model on deviation from maximum return is *a*, the weight on deviation from maximum skewness is *b*.

TABLE 4: Polynomial goal programming: all markets

	Daily Returns						Weekly Returns						Monthly Returns					
Parameters																		
a	1	0	1	2	1	2	1	0	1	2	1	2	1	0	1	2	1	2
b	0	1	1	1	2	2	0	1	1	1	2	2	0	1	1	1	2	2
Optimal Portfolio Composition																		
ARGENTINA	2.74%	0.29%	2.86%															
AUSTRALIA	2.08%						4.27%						1.75%					
AUSTRIA	1.67%			0.44%			8.41%						3.18%	4.70%	5.73%	4.46%		11.42%
BELGIUM	2.21%			0.31%	1.65%		2.03%							5.74%	6.51%	6.53%	1.69%	3.66%
BRAZIL	3.13%							2.21%	2.36%	2.37%				4.31%	4.56%	4.44%		5.80%
CANADA	1.94%						13.23%						8.97%					
CHILE	1.86%							1.73%	1.94%	2.07%								
CHINA	0.89%				0.82%	1.71%	10.98%	77.00%	76.92%	77.54%			8.94%	80.54%	77.92%	79.67%	74.83%	68.20%
DENMARK	2.15%						18.62%						35.82%				13.52%	
FINLAND	4.60%						3.26%											0.45%
FRANCE	3.05%																	
GERMANY	3.10%																	
GREECE	2.55%					0.20%	5.93%	3.90%	4.15%	4.01%			3.58%					
HONG KONG	3.48%	2.95%	2.49%	3.35%	6.01%	4.93%												
INDONESIA	3.94%	3.63%	1.68%	2.11%	13.92%	15.67%					48.06%	48.19%						
IRELAND	2.17%	1.54%			1.98%		16.10%						12.94%	1.89%	2.71%	2.12%		9.45%
ITALY	2.85%																	
JAPAN	2.15%																	
KOREA	3.99%				2.25%						14.12%	14.16%						
MALAYSIA	2.64%							0.79%	0.68%	0.71%								
MEXICO	3.07%							2.53%	2.32%	2.35%								
NETHERLANDS	2.99%																	
NEW ZEALAND	1.78%						2.25%	2.02%	2.01%	1.82%								1.01%
NORWAY	2.76%																	
PHILIPPINES	2.17%	20.61%	21.97%	21.90%	14.10%	15.71%											5.70%	
POLAND	3.34%							0.11%	0.09%	0.09%								
PORTUGAL	2.07%	1.19%		0.11%	0.68%	0.01%		3.12%	3.09%	3.18%	20.43%	20.29%						
SINGAPORE	2.68%																4.26%	
SPAIN	2.95%		0.06%		1.20%	0.27%	0.73%											
SWEDEN	3.83%																	
SWITZERLAND	2.41%										5.11%	4.96%						
TAIWAN	2.18%																	
THAILAND	3.72%	62.87%	63.60%	64.33%	49.82%	55.24%					12.28%	12.41%		2.21%	1.63%	2.02%		
TURKEY	4.62%						4.79%						1.94%					
UK	2.38%	0.17%	0.54%		1.54%	0.63%		6.58%	6.44%	5.85%				0.61%	0.94%	0.76%		
US	1.57%	3.53%	4.51%	4.22%	6.04%	4.39%	9.39%						22.89%					
VENEZUELA	2.26%	3.23%	2.30%	3.15%		1.32%												
Optimal portfolio statistics (all are unit variance)																		
mean	4.75%	0.65%	0.66%	0.63%	0.66%	0.66%	14.66%	6.46%	6.47%	6.44%	2.90%	2.89%	30.43%	12.72%	12.96%	12.80%	12.93%	14.05%
skewness	-0.38	0.81	0.81	0.81	0.79	0.80	0.04	8.62	8.62	8.62	1.12	1.12	-0.42	6.66	6.66	6.66	6.56	6.63

Note: The weight in the goal programming model on deviation from maximum return is *a*, the weight on deviation from maximum skewness is b.

Endnotes:

[1] See Harris, Kucukozmen, and Yilmaz (2004), p. 195.

[2] See Josey, Brooks and Faff (2001). To avoid any confusion with phrases like "invest horizon", "holding period rate of return" etc. we have adopted the phrase "investment interval" throughout this paper.

[3] This brief discussion on the effect of intervalling on variance and skewness of the rates of return probability distribution is largely taken from Prakash, de Boyrie, Hamid and Smyser (1997).

[4] Dividends are ignored. Whether they are deterministic or random they can be considered a part of

P_{j-1} or $\tilde{P}_j$ respectively.

[5] Whether we examine the probability distribution of $\tilde{r}_j$ or one plus the rate of return (wealth ratio) the result remains the same.

[6] Some of the skewness measures are negative. Here, we provide the count for absolute skewness only. If, however, we do not ignore the negative signs, then the weekly skewness in two, rather than one, instances is greater than its monthly counterparts.

[7] To convert the standard deviation of the annualized weekly return to the weekly return, divide the obtained standard deviation by the square root of 52. Similarly, the monthly standard deviation can be obtained by dividing the standard deviation of the annualized monthly return by the square root of 12.

[8] There will not be any difference in the measure of skewness whether it is obtained "holding period" or nominally annualized returns.

[9] See Prakash et. al. (2003) for exact definitions of *a* and *b*.

[10] Other combination values of *a* and *b* is provided because in this programming problem one can increase the preferences for a parameter at will. However, theoretically speaking, the paper is concerned mainly with the comparison of mean-variance versus mean-variance-skewness preferences.

[11] We are indebted to an anonymous referee for pointing this findings out. The explanations have been provided about these seemingly "implausible" findings.