L10

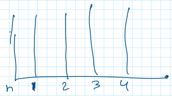
Monday, February 22, 2021 4:58 PM

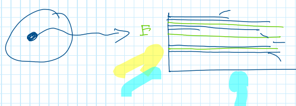
SPIN

Atom

Abraham Pais

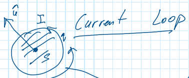
Inward Bound





- Splitting of lines
- number of lines are odd

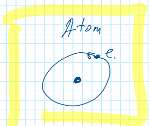
Classical



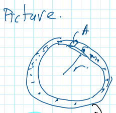
$$\boxed{\vec{\mu} = I \cdot S\hat{u}}$$

$$V = -\vec{\mu}\cdot\vec{B}$$

Classical physics



microscopic Picture.



$I = j \cdot A$

$j = n\,q \cdot V_d$

$$\vec{\mu} = \frac{nq\,V \cdot A \cdot S}{2\pi r}\cdot \pi r^2 = \frac{Q \cdot V \cdot S}{2\pi r}\hat{u}$$

$$Q = -e = q$$

$$\boxed{\vec{\mu} = \frac{Q\,V \cdot \pi r^2}{2\pi r}\hat{u} = \frac{Q \cdot V \cdot r}{2}\hat{u}}$$

$$\vec{\mu} = \frac{q\,mV \cdot r}{2m}\hat{u} =$$

$$= \frac{q\,p \cdot r\,\hat{u}}{2m} = \frac{q\,(r \cdot p \cdot \hat{u})}{2m} = \frac{q\,\vec{L}}{2m}$$

$\vec{L} = \vec{r} \times \vec{p}$

$$\boxed{\vec{\mu} = \frac{q\,\vec{L}}{2m}\hbar}$$

electron

$$\boxed{\vec{\mu}_e = -\frac{e\,\vec{L}\,\hbar}{2m}}$$

⇒ Magnetic Field $\boxed{B}$ $\boxed{V_B = -\vec{\mu}\cdot\vec{B}}$

⇒ Hydrogen Atom No B

$$\hat{H} = \frac{p^2}{2m} - \frac{Ze^2}{r} = \frac{p^2}{2m} + V_c(r)$$

⇒ Magnetic Field

$$\hat{H} = \frac{p^2}{2m} + V_c(r) + \boxed{V_B} =$$

$$\frac{p^2}{2m} + V_c(r) - \vec{\mu}\cdot\vec{B} =$$

$$\hat{H} = \frac{p^2}{2m} - \frac{Ze^2}{r} + \frac{e\,\vec{L}\cdot\vec{B}}{2m_e}$$

⇒ $\vec{B}$ in z direction

$$\hat{H} = \frac{p^2}{2m} - \frac{Ze^2}{r} + \frac{e\,L_z \cdot B_z}{2m_e}$$

$$\hat{H}|\Psi_{Bem}\rangle = E|\Psi_{Bem}\rangle$$

$$\left(\frac{p^2}{2m} - \frac{Ze^2}{r} + e\frac{L_z\,B_z}{2m_e}\right)\Psi_{Bem}(r,\theta,\phi) = E\,\Psi_{Bem}(r,\theta,\phi)$$

$\to R_{Be}(r)\,Y_e^m(\theta,\phi)$

$$p^2 = -\hbar^2\nabla^2 = -\hbar^2\left[\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr} - \frac{\hat{L}^2}{r^2}\right]$$

$e(e+1)$

$$\left(-\frac{\hbar^2}{2m}\left(\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr} - \frac{\hat{L}^2}{r^2}\right) - \frac{Ze^2}{r} + e\frac{L_z\,B_z}{2m_e}\right)R\,Y_{em}(\theta,\phi) = E\,R\,Y_e^m(\theta,\phi)$$

$$L_z Y_e^m(\theta,\phi) = m\,Y_e^m(\theta,\phi)$$

$$\Rightarrow \left[\left(-\frac{\hbar^2}{2m}\left(\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr} - \frac{e(e+1)}{r^2}\right) - \frac{Ze^2}{r} + e\frac{m\,B_z}{2m_e}\right]R_{Bem} = E_{n,m}\,R_{Be}$$

$n = (2j+1)$

j - contained e.

$e = 1$ $\quad m \to -1,\ 0,\ 1$

$$-e \le m \le e$$

l ← Intege

$\boxed{(2l+1)} - m$ — odd.

splitting line numbers are odd

Zeeman Effect

No splitting for ground state

Balmer Formula
1885

\lambda = h (m^2/(m^2-n^2)

1/lambda = R_H(1/n_1^2 - 1/n_2^2)

1896 Pieter Zeeman

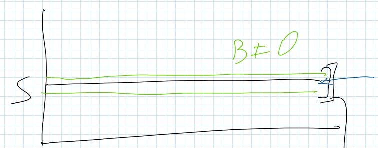
– Experiment will show even number for splitting.

– Anomaleous Zeeman Effect.

– S – state $l = 0$

splits into two lines

$l=0$ S
$l=1$ P
$l=2$ D



$l = ?$

$l = \frac{1}{2}$

m $2l+1$

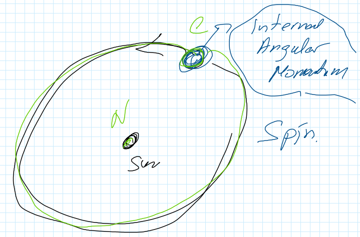
$\boxed{m = 2\cdot\frac{1}{2}+1 = 2}$

$\vec{L}$ – Orbital Angular Momentum

? $\vec{S}$ – Internal Angular Momentum

Spin.

$\boxed{S = \frac{1}{2}.}$



⟹ Generator of Rotation $\boxed{\hat{J}_i}$

$J^2|\psi_{j,j_z}\rangle = j(j+1)|\psi_{j,j_z}\rangle$

j
→ integer
→ half integer → $\frac{1}{2}$

⟹ Generator of Rotations J_i

– $[J_i, J_j] = i\sum_k \varepsilon_{ijk} J_k$!

– $J^2 = J_x^2 + J_y^2 + J_z^2$ – Kazimir Operator

– $[J_i, J^2] = 0$

– $J_\pm = J_x \pm i J_y$

J^2, J_z

– $|\psi_{j,m}\rangle$ → $[J^2, J_z] = 0$

– $J_z|\psi_{jm}\rangle = m|\psi_{jm}\rangle$

$\hat{J}_\pm = J_x \pm i J_y$

– $J^2|\psi_{jm}\rangle = j(j+1)|\psi_{jm}\rangle$

$- \hat{J}^2 |\psi_{jm}\rangle = j(j+1)|\psi_{jm}\rangle$

$- J_\pm |\psi_{jm}\rangle = \boxed{\sqrt{j(j+1) - m(m\pm 1)}}\, |\psi_{j,m\pm 1}\rangle$

$\Rightarrow$ Spin $\boxed{\frac{1}{2}}$ $\quad \boxed{\hat{J}_i} = \boxed{\hat{S}_i} \quad \boxed{\begin{matrix} |\psi_{s,m_s}\rangle \\ |s, m_s\rangle \end{matrix}}$

- $[\hat{S}_i \hat{S}_j] = i \sum_k \varepsilon_{ijk} \hat{S}_k$
- $S^2 = S_x^2 + S_y^2 + S_z^2$
- $[\hat{S}_i \hat{S}^2] = 0$
- $\hat{S}_z |s\ m_s\rangle = m_s |s, m_s\rangle$
- $\hat{S}^2 |s, m_s\rangle = s(s+1) |s, m_s\rangle$, $\quad \boxed{s = m_s^{max}}$
- $S_\pm |s, m_s\rangle = \sqrt{s(s+1) - m_s(m_s \pm 1)}\, |s, m_s \pm 1\rangle$

$\boxed{\hat{S}_i =}$? $\quad \boxed{|s, m_s\rangle}$?