L25

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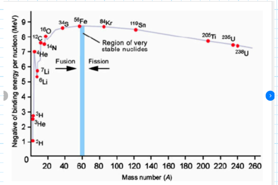
White Dwarf Stars



Binding energy per nucleon B (Z , N) /A as a function of the mass number (

- Energy of star is from Fusion

$H + H \to D + \beta^{+} + \nu \qquad Q = 1.44\ MeV$

$H + D \to {}^{3}He + \gamma, \qquad Q = 5.43\ MeV$

${}^{3}He + {}^{3}He \to {}^{4}He \to 2H \qquad 12.86\ MeV$

${}^{2}H + {}^{3}H \to {}^{4}He + 3.5\ MeV$
$n + 14.1\ MeV$

- Then Thermonuclear Reaction Stops
- Stars Cool Down – Thermal Pressure Decreases
- Star Begins to Collapse
- Collapse heats up the star – Fusing lighter Elements to Heavy Element Untill Fe
- For Fe – No more Nuclear Reaction
- Cooling
- Gravitational Collapse takes over one more
- Only Degeneracy Pressure Keeps star from fully collapsing.

$e \to$ generating Fermi Pressure. Non Relativistic Case

$e \to$ Relativistic. — Chandrasecar Limit.
$P_{e} \to n$ — Fermi Pressure of neutrons $\to$ Neutron star. Black – Holes

- Let's Introduce uniform, spherical star with R and $\boxed{M}$
- Electrons in the star generate Fermi Pressure

$$\boxed{P_{press.} = \frac{p_F^5}{15\pi^2 m_e}}$$

- Mass of Proton and Neutron (Nucleon) is 2000 times more than mass of e. Therefore $\frac{M_{star}}{m_N} = \boxed{N}$ – number of nucleons

Neutral.
- Number of e. $N_e = \frac{N}{2}$ 4 $Z = \frac{A}{2}$

in general $N_e = \frac{M}{\mu m_N}$, $\mu = \frac{A}{Z} = \frac{56}{26}$

$(3\pi^2 \rho)^{\frac{1}{3}}$

$$- p_F^{e} = \left(3\pi^2 \frac{N_e}{V}\right)^{\frac{1}{3}} = \left(\frac{3\pi^2}{\frac{4\pi}{3}R^3}\frac{M}{\mu m_N}\right)^{\frac{1}{3}} = \left(\frac{9\pi}{4}\frac{M}{\mu m_N}\right)^{\frac{1}{3}} \cdot \frac{1}{R}$$

$\boxed{V = \frac{4}{3}\pi R^3}$ $\quad N_e = \frac{M}{\mu m_N}$

$$- \text{Pressure} \quad P_e = \frac{p_F^5}{15\pi^2 m_e} = \frac{(3\pi^2)^{\frac{2}{3}}}{5 m_e R^5}\left(\frac{3M}{4\pi \mu m_N}\right)^{5/3}$$

$\Rightarrow$ Consider a spherical Uniform Star $\qquad M(r) = \frac{4\pi}{3} r^3 \cdot \rho_{mass}$

$dm =$

- Gravitational Potential Energy

$$W(R) \quad U(R) = -\int_0^R \frac{G M(r)\, dm}{r} = -\int_0^R \frac{G \frac{4\pi}{3} r^3}{r} d^3r \cdot \rho^2$$

ρ = mass density = const

$dm = \rho_{mass} d^3r$

$M(r)$

$$U = -\rho^2 \frac{4\pi G}{3}\int_0^R r^2 d^3r = -\rho^2 \frac{4\pi G}{3}\int_0^R r^2 r^2 dr\, d\Omega$$

$$W(R) = -\rho^2 \frac{(4\pi)^2 G}{3}\frac{r^5}{5}\Big|_0^R = -\frac{3}{5}\frac{GM^2}{R} = -\frac{3}{5}GM^2\left(\frac{4\pi}{3V}\right)^{\frac{1}{3}}$$

$V = \frac{4\pi}{3}R^3$

$$P_G = -\frac{dW}{dV} = \frac{1}{3}\frac{3}{5}GM^2\left(\frac{4\pi}{3}\right)^{\frac{1}{3}}V^{-\frac{4}{3}} = \frac{3}{5}\frac{GM^2}{4\pi}\frac{1}{R^4}$$

$$\left(\frac{4\pi}{3}\right)^{3}\frac{1}{\left(\frac{4\pi}{3}\right)^{\frac{4}{3}}R^4} = \frac{1}{\left(\frac{4\pi}{3}\right)R^4}$$

$\Rightarrow$ We have

$$\boxed{P_e} = \frac{1}{5m_e R^5}(3\pi^2)^{2/3}\left(\frac{3M}{4\pi\mu m_N}\right)^{5/3} \; !$$

$$\boxed{P_G} = \frac{3}{4\pi}\frac{GM^2}{5R^4}$$

$\Rightarrow$ When star collapses $\quad P_G \uparrow \frac{1}{R^4}$, $\quad P_e \uparrow \frac{1}{R^5}$

- Fermi Gas will overhelm Gravitational Pressure

- At some R Two pressures will Balance each other

$$\boxed{P_G = P_e} \quad (\text{solve for } R)$$

$$R = \frac{1}{G m_e m_N^2}\left(\frac{9\pi}{4}\right)^{2/3}\left(\frac{1}{\mu}\right)^{5/3}\left(\frac{m_N}{M}\right)^{\frac{1}{3}}$$

$\Rightarrow M = x \cdot M_s \qquad M_s = 1.98\times10^{33}g, \quad \mu = 2$

$$R = \frac{7.18\times10^8\,cm}{x^{\frac{1}{3}}} \underset{x=1}{=} 7,180\,km$$

$R_E = 6400\,km$

$$\rho = \frac{3M}{4\pi R^3} \simeq 1.3\times10^7 \cdot x^2 \frac{g}{cm^3}$$

$$\rho_{sun}^{today} = 1.4\frac{g}{cm^3}$$

$$\rho_{Earth} \simeq 5.5\frac{g}{cm^3}$$

$$\hookrightarrow E_f = \frac{1}{2m}\left(3\pi^2\frac{N}{V}\right)^{2/3} = 2\times10^5\,x^{4/3}\,eV = 0.2\,x^{4/3}\,MeV$$

(Electrons Become Almost Relativistic)

$x = 1$ sun

$E_f = 0.2\,MeV$

$m_e = 0.52\,MeV$

$\Rightarrow$ Relativistic Electrons

More Massive Stars

$$E_F = \sqrt{m_e^2 + P_F^2}, \qquad E_F = \frac{P_F^2}{2m} \quad N.R.$$

$$E = 2V\int_0^{P_F}\sqrt{p^2+m^2}\,\frac{d^3p}{(2\pi)^3} = m^3\,2V\int_0^{P_F} m\sqrt{1+\frac{p^2}{m^2}}\,\frac{p^2}{m^2}\frac{dp}{m}\frac{d\Omega}{8\pi^3}^{4\pi} \qquad t = \frac{p}{m}$$

$$\boxed{E_{TOT} = \frac{m^4 V}{\pi^2}F(x_F)} \qquad \boxed{F(x_F) = \int_0^{x_F} t^2\sqrt{1+t^2}\,dt} \qquad \boxed{x_F = \frac{P_F}{m}}$$

$$P = -\frac{\partial E}{\partial V} = -\frac{m^4}{\pi^2}\left[F(x_F) - \frac{1}{3}x_F^3\sqrt{1+x_F^2}\right]$$

$$V\frac{\partial F(x_F)}{\partial V} = V x_F^2\sqrt{1+x_F^2}\,\frac{\partial x_F}{\partial V} = V x_F^2\sqrt{1+x_F^2}\,\frac{1}{m}\frac{\partial P_F}{\partial V} = V x_F^2\sqrt{1+x_F^2}\left(-\frac{1}{3}\frac{P_F}{m}\right)\frac{1}{V}$$

$$P_F = \left(3\pi^2\frac{N_e}{V}\right)^{\frac{1}{3}}$$

$$\frac{\partial P_F}{\partial V} = -\frac{1}{3}(3\pi^2 N_e)^{\frac{1}{3}}V^{-\frac{4}{3}} = -\frac{1}{3}\frac{\left(3\pi^2\frac{N_e}{V}\right)^{\frac{1}{3}}}{V} = -\frac{1}{3}\frac{P_F}{V}$$

Ultrarelativistic case $\boxed{x_F >> 1}$ [Show]

$$x_F\sqrt{1+\frac{1}{x_F^2}}$$

$$P = +\frac{m_e^4}{12\pi^2}\left[\underset{keep}{x_F^4} + \underset{drop}{x_F^2}\right]$$

$$x_F = \frac{P_F}{m_e} \qquad P_F = \left(3\pi^2\frac{N}{V}\right)^{\frac{1}{3}}$$

$$\boxed{P_{Frm} = \frac{1}{12\pi^2}\frac{1}{R^4}\left(\frac{3\pi M}{4 m_N \mu}\right)^{4/3}} \oplus \qquad P_{Frm} \uparrow \frac{1}{R^4}$$

$x_F >> 1$

$$F(x_F) \simeq \int_0^{x_F} t^3\sqrt{1+\frac{1}{t^2}}\,dt$$

$$\int_0^{x_F}\left(t^3 + \frac{t}{2}\right)dt \simeq \frac{x_F^4}{4}$$

$$x_F^4\left(\frac{1}{4} - \frac{1}{3}\right) = -\frac{x_F^4}{12}$$

$$P_G = \frac{3}{4\pi}\frac{GM^2}{5R^4} \qquad P_G \uparrow \frac{1}{R^4}$$

if $P_G \leq P_{Fermi}$

$P_G \geq P_{Ferm}$

$\boxed{P_G \leq P_{Ferme}}$ star will not collapse

P-drop out.

$X_F = \frac{P_F}{m_e} = \frac{1}{m_e}\left(\frac{9}{4}\pi\frac{M}{\mu m_N}\right)^{\frac{1}{3}}\frac{1}{R}$

Solve For $\boxed{M_{star}}$

$M_{star} = 1.4\ M_\odot$

$\Rightarrow$ Critical Mass $P_{Fermi} = P_G$ and calculate M_c