Homework 1

1. (20 points) Starting with the algebra of rotational generators

$$\left[\hat{J}_i\ \hat{J}_j\right] = i \sum_k \epsilon_{ijk}\ \hat{J}_k$$

show that the following relations are *true* :

$$\left[\hat{J}^2, \hat{J}_j\right] = 0.$$

$$\left[\hat{J}_z, \hat{J}_\pm\right] = \pm\hat{J}_\pm$$

$$\left[\hat{J}_+, \hat{J}_-\right] = 2\,\hat{J}_z$$

$$\left[\hat{J}_\pm, \hat{J}^2\right] = 0$$

2. (20 points) Show that $\hat{J}_+$ and $\hat{J}_-$ represent step up and step down operators for the eigenstate of the operator $\hat{J}_z$. Calculate the eigenvalue of $\hat{J}^2$ operators.

3. (20 points) Show that $\hat{J}_\pm \left| \psi_{j,m} \right> = \sqrt{j\ (j+1) - m\ (m \pm 1)}\ \left|\psi_{j,m\pm1}\right>$

4. (20 points) Calculate the matrix elements of $\hat{J}_\pm$ and $\hat{J}_z$ operators.
   That is $\left< \psi_{j,m'} \right| J_\pm \left| \psi_{j,m} \right>$ and $\left< \psi_{j,m'} \right| J_z \left| \psi_{j,m} \right>$

5. (50 points) Show that $\left[\hat{p}_i, \hat{L}_j\right] = i \sum_k \epsilon_{ijk}\ \hat{p}_k$ and $\left[\hat{r}_i, \hat{L}_j\right] = i \sum_k \epsilon_{ijk}\ \hat{r}_k$

   - Show explicitely that these generators satisfy the following commutator *relations* : $\left[\hat{L}_i\ \hat{L}_j\right] = i \sum_k \epsilon_{ijk}\ \hat{L}_k$

6. (70 points) Obtain the expressions for the operators $\hat{L}^2$, $\hat{L}_x$, $\hat{L}_y$ and $\hat{L}_z$ in polar coordinates as it acts on a wave function.