

Homework 5

1. (40 points) Obtain the explicit forms of the generators of rotation in spin half space. Obtain also the eigenstates and eigenvalues of S^2 and S_z as well as S_x and S_y operators.

2. (40 points) For Pauli matrices prove the following *relations* :

$$[\sigma_i, \sigma_j] = 2i \sum_k \epsilon_{ijk} \sigma_k$$

$$\sigma_i = \sigma_i^\dagger = \sigma_i^{-1}$$

$$\det \sigma_i = -1$$

$$(\hat{n} \cdot \vec{\sigma})^2 = 1$$

$$\sigma_i \sigma_j + \sigma_j \sigma_i = 2 \delta_{ij}$$

$$(\sigma_i)^2 = 1$$

$$\text{Tr} \sigma_i = 0$$

$$\sigma_i \sigma_j = \delta_{ij} + i \sum_k \epsilon_{ijk} \sigma_k$$

3. (30 points) Show that $e^{-i \frac{1}{2} \mathbf{n} \cdot \boldsymbol{\sigma} \theta} = \mathbf{I} \cos(\theta/2) - i \mathbf{n} \cdot \boldsymbol{\sigma} \sin(\theta/2)$
Calculate the rotational operators for the rotation around x, y and z axes.

4. (40 points) Present the brief description of Stern - Gerlach experiment.