```
(*Covariant form of the g_ij in FLRW metric *)
g4cov[i_, j_, r_, θ_, ϕ_, t_] := Module[{f = 0},
   If[i == 0 && j == 0,  f =  1];

   If[i == 1 && j == 1,  f =  - a[t]^2/(1 - k r^2)];
   If[i == 2 && j == 2,  f =  -a[t]^2 r^2];
   If[i == 3 && j == 3,  f =  -a[t]^2 r^2 (Sin[θ])^2];
   f];

(*Contravariant form of the g^ij in FLRW metric *)
g4cont[i_, j_, r_, θ_, ϕ_, t_] := Module[{f = 0},
   If[i == 0 && j == 0,  f =  1];

   If[i == 1 && j == 1,  f =  - (a[t]^2/(1 - k r^2))^-1];
   If[i == 2 && j == 2,  f =  -(a[t]^2 r^2)^-1];
   If[i == 3 && j == 3,  f =  -(a[t]^2 r^2 (Sin[θ])^2)^-1];
   f];

(* dvar[μ_]:= Module[{f=0},
   If[μ==0,f=t];
   If[μ==1,f=r];
   If[μ==2,f=θ];
   If[μ==3,f=ϕ];
   f]; *)

(*dvar[3]
 D[g4cov[1,1,r,θ,ϕ,t],dvar[0]] *)


(* g_ij,l *)
dg4cov[i_, j_, l_, r_, θ_, ϕ_, t_] := Module[{f = 0},
   If[l == 0,  f =  1/c D[g4cov[i, j, r, θ, ϕ, t], t]];
   If[l == 1,  f = D[g4cov[i, j, r, θ, ϕ, t], r]];
   If[l == 2,  f = D[g4cov[i, j, r, θ, ϕ, t], θ]];
   If[l == 3,  f = D[g4cov[i, j, r, θ, ϕ, t], ϕ]];
   f];
```

In[9]:=
```
dg4cov[1, 1, 0, r, θ, ϕ, t]
```

Out[9]=
$$-\frac{2\, a[t]\, a'[t]}{c\,\left(1-k\,r^2\right)}$$

```
(* Γ^λ_μν *)
Γ4[μ_, ν_, λ_, r_, θ_, ϕ_, t_] := Module[{f = 0, k = 0},
   f = Sum[g4cont[λ, k, r, θ, ϕ, t] ×
       (dg4cov[k, ν, μ, r, θ, ϕ, t] +
         dg4cov[k, μ, ν, r, θ, ϕ, t] - dg4cov[μ, ν, k, r, θ, ϕ, t]), {k, 0, 3}];
   (*(D[g4cov[k,ν,r,θ,ϕ,t],dvar[μ]] + D[g4cov[k,μ,r,θ,ϕ,t],dvar[ν]] -
      D[g4cov[μ,ν,r,θ,ϕ,t],dvar[k]]); *)
   1/2
    f];
```

In[12]:=
```
Γ4[1, 1, 0, r, θ, ϕ, t]
```

Out[12]=
$$\frac{a[t]\, a'[t]}{c\,\left(1-k\,r^2\right)}$$

```
(*Γ^k_ij,l*)
dΓ4[i_, j_, k_, l_, r_, θ_, ϕ_, t_] := Module[{f = 0},
   If[l == 0, f = 1/c D[Γ4[i, j, k, r, θ, ϕ, t], t]];
   If[l == 1, f = D[Γ4[i, j, k, r, θ, ϕ, t], r]];
   If[l == 2, f = D[Γ4[i, j, k, r, θ, ϕ, t], θ]];
   If[l == 3, f = D[Γ4[i, j, k, r, θ, ϕ, t], ϕ]];
   f];

(*Reiman Tensor R
   Reiman4[i_,j_,k_,l_,r_,θ_,ϕ_,t_]:= Module[{f=0},
   f = dΓ4[i,k,l,j,r,θ,ϕ,t]- dΓ4[i,j,l,k,r,θ,ϕ,t] +
     Σ_{al=0}^3(Γ4[i,k,al,r,θ,ϕ,t]Γ4[al,j,l,r,θ,ϕ,t]-
        Γ4[i,j,al,r,θ,ϕ,t]Γ4[al,k,l,r,θ,ϕ,t]);
   f];
```

In[19]:=
```
Ricci4[i_, j_, r_, θ_, ϕ_, t_] := Sum[Reiman4[i, j, l, l, r, θ, ϕ, t], {l, 0, 3}];
Simplify[Ricci4[0, 0, r, θ, ϕ, t]]
```

$$\sum_{l=0}^{3}$$

Out[20]=

$$\frac{3\, a''[t]}{c^2\, a[t]}$$

In[21]:=
```
R4 = Simplify[Sum[g4cont[i, i, r, θ, ϕ, t] × Ricci4[i, i, r, θ, ϕ, t], {i, 0, 3}]]
```

Out[21]=

$$\frac{6\left(c^2\, k + a'[t]^2 + a[t]\, a''[t]\right)}{c^2\, a[t]^2}$$

In[31]:=
```
G0u0d = Simplify[g4cont[0, 0, r, θ, ϕ, t] × Ricci4[0, 0, r, θ, ϕ, t] - 1/2 R4]
```

Out[31]=

$$-\frac{3\left(c^2\, k + a'[t]^2\right)}{c^2\, a[t]^2}$$

In[32]:=
```
G1u1d = Simplify[g4cont[1, 1, r, θ, ϕ, t] × Ricci4[1, 1, r, θ, ϕ, t] - 1/2 R4]
```

Out[32]=

$$-\frac{c^2\, k + a'[t]^2 + 2\, a[t]\, a''[t]}{c^2\, a[t]^2}$$

In[33]:=
```
G22 = Simplify[g4cont[2, 2, r, θ, ϕ, t] × Ricci4[2, 2, r, θ, ϕ, t] - 1/2 R4]
```

Out[33]=

$$-\frac{c^2\, k + a'[t]^2 + 2\, a[t]\, a''[t]}{c^2\, a[t]^2}$$

In[34]:=
```
G33 = Simplify[g4cont[3, 3, r, θ, ϕ, t] × Ricci4[3, 3, r, θ, ϕ, t] - 1/2 R4]
```

Out[34]=

$$-\frac{c^2\, k + a'[t]^2 + 2\, a[t]\, a''[t]}{c^2\, a[t]^2}$$