Exam 1

1. (10 points)

(a) Prove that if $\lim_{n\to\infty} n^p u_n = A$ - finite, $p > 1$,

the series $\sum_{n=1}^{\infty} u_n$ converges

(b) Prove that if $\lim_{n\to\infty} n \cdot u_n = A > 0$, the series $\sum_{n=1}^{\infty} u_n$ diverge

2. (10 points) With $n > 1$ show that

(a) $\frac{1}{n} - \ln\left(\frac{n}{n-1}\right) < 0$,

(b) $\frac{1}{n} - \ln\left(\frac{n+1}{n}\right) > 0$

3. (10 points) Evaluate $\lim_{x\to 0}\left[\frac{\sin(\tan x) - \tan(\sin(x))}{x^7}\right]$

4. (10 points) Expand function $P(x) = c\left(\frac{\cosh(x)}{\sinh(x)} - \frac{1}{x}\right)$

as a power series for small x

5. (15 points) Show that rotation does not change the scalar product of vectors

(consider 2 dimensional case)

6. (15 points) Using Levi - Civita constants

calculate $\vec{A}\times\left(\vec{B}\times\vec{C}\right)$, $\vec{A}\times\left(\vec{\nabla}\times\vec{C}\right)$, $\vec{\nabla}\times\left(\vec{B}\times\vec{C}\right)$ and $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{V}\right)$

7. (15 points) Start with Maxwell equations with Electric and Magnetic fields and express them through field potentials

8. (15 points) Express $\frac{\partial}{\partial x}$, $\frac{\partial}{\partial y}$ and $\frac{\partial}{\partial z}$ in sperical polar coordinates.