Homework 9 (20 points each)

1. Show that for two complex numbers z_1 and z_2

$$||z_1| - |z_2|| \le |z_1 \pm z_2| \le |z_1| + |z_2|$$

2. Calculate the following functions

$\sin^{-1}(z)$, $\sinh^{-1}(z)$

$\tan^{-1}(z)$, $\tanh^{-1}(z)$

$\cos^{-1}(z)$, $\cos^{-1}(z)$

3. Prove that

$$\sum_{n=0}^{N-1} \cos(nx) = \frac{\sin(Nx/2)}{\sin(x/2)} \cos(N-1)\frac{x}{2}$$

4. Find the analytic function

$w(z) = u(x, y) + i\,v(x, y)$

if

(a) $u(x, y) = x^3 - 3xy^2$

(b) $u(x, y) = e^{-y}\cos(x)$

5. For following $f(z)$ functions calculate $f'(z)$ and identify the maximal region within which $f(z)$ is analytic

(a) $f(z) = \dfrac{\sin(z)}{z}$

(b) $f(z) = \dfrac{1}{z(z+1)}$

(c) $f(z) = \tan(z)$

(d) $f(z) = e^{-1/z}$