Ginvariance

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- In the handbook

$$A'_\mu = S A_\mu S^{-1} + \frac{i}{g} [\partial_\mu S] S^{-1}$$

$A'^b_\mu t^b$

$$(1 + i w_a t_a) A^b_\mu t^b (1 - i w_a t_a) + \frac{i}{g} [\partial_\mu i w_a t_a] [1 - i w_a t_a]$$

$$= A^b_\mu t_b + i w_a T_a A_\mu^{t_b} - A_\mu^{t_b} i w_a t_a + \frac{i}{g} [\partial_\mu i w t_a] [1 - i w t_a]$$

$$= A_\mu^b t_b + i w_a A_\mu^b [t_a t^b - t^b t^a] - \frac{1}{g} \partial_\mu w t_a$$

$$= A_\mu^b t_b - w_a A_\mu^b f^{abc} t^c - \frac{1}{g} \partial_\mu w t_a$$

$$A'^b_\mu t^b = A_\mu^b t_b - w_a A_\mu^b f^{abc} t^c - \frac{1}{g} \partial_\mu w t_a$$

$\Rightarrow$ Consider $-\frac{1}{4} F^{\mu\nu}_a F^{\mu\nu}_a =$

$$= -\frac{Tr}{2} \left((F^{\mu\nu}_a t_a) F^{\mu\nu}_b t_b \right)$$

using the fact that $Tr\, t_a t_b = \frac{1}{2} \delta^{ab}$

$\Rightarrow$ Now we consider the Gauge Transformation of:

$F'^{\mu\nu}$ using $A'^a = A^a - \frac{1}{g} \partial w_a$

$\hookrightarrow t_a^{\mu\nu} t_a$ using $A'_\mu = A_\mu - \frac{1}{g}\partial_\mu \omega$
$- w_b A_\mu^c f^{abc}$

$$F_a'^{\mu\nu} t_a = \partial_\mu A_a'^\nu - \partial_\nu A_a'^\mu - g f^{abc} A'_{\mu b} A'_{\nu c} =$$

need do slow

$$\partial_\mu \hat{A}^\nu - \partial_\nu \hat{A}^\mu - g f^{abc} A_{\mu b} A_{\nu c} t^a +$$
$$+ i\hat{w}(\partial_\mu \hat{A}^\nu - \partial_\nu \hat{A}^\mu) - (\partial_\mu \hat{A}^\nu - \partial_\nu \hat{A}^\mu) i\hat{w}$$
$$- i\hat{w} g f^{abe} A_{\mu b} A_{\nu c} t^a + i g f^{abe} A_{\mu b} A_{\nu c} t^a \hat{w} =$$

$$\partial_\mu \hat{A}_\nu - \frac{1}{g}\partial_\nu \partial_\mu \hat{w} - \partial_\mu (w_b A_\nu^c t^a f^{abc})$$
$$-\left(\partial_\nu \hat{A}_\mu - \frac{1}{g}\partial_\nu \partial_\mu \hat{w} - \partial_\nu (w_b A_\mu^c t^a f^{abc})\right)$$
$$- g f^{abc} t^a \left(A_\mu^b - \frac{1}{g}\partial_\mu w^b - w_b' A_\mu^{c'} f^{bb'c'}\right)$$
$$\left(A_\nu^c - \frac{1}{g}\partial_\nu w^c - w_b' A_\nu^{c'} f^{cb'c'}\right) =$$

w^0

$$= \boxed{\partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu - g t^{abe} f^a A_\mu^b A_\nu^c}$$
$$- (\partial_\mu w_b) A_\nu^c t^a f^{abe} - w_b \partial_\mu A_\nu^c t^a f^{abe}$$
$$- (-\partial_\nu w_b) A_\mu^c t^a f^{abe} - w_b \partial_\nu A_\mu^c t^a f^{abc})$$
(a) (c) (b) (c)

$$-gf^{abc}t^a A_\mu^b\left(-\frac{1}{g}\partial_\nu w^c\right) - gf^{abc}t^a A_\mu^b\left(-w_{b'}A_\nu^{c'}f^{cb'c'}\right)$$

(b) (d)

$$-gf^{abc}t^a\left(-\frac{1}{g}\partial_\mu w^b\right)A_\nu^c - gf^{abc}t^a\left(-w_{b'}A_\mu^{c'}f^{bb'c'}\right)A_\nu^c$$

(a) (d')

$$(a+a') = 0$$

$$(b+b') = \cancel{\partial_\nu w^b A_\mu^c t^a f^{abc}} + \cancel{A_\nu^b \partial_\nu w^c f^{abc} t^a} = 0$$

$$(c+c') = -w_b(\partial_\mu A_\nu^c - \partial A_\mu^c)t^a f^{abc} = \text{"}$$

$$[t^b t^c - t^c t^b] = if^{abc}t^a$$

$$\text{"}\; i\, w^b(\partial_\mu A_\nu^c - \partial_\nu A_\mu^c)[t^b t^c - t^c t^b] =$$

$$i[\hat{w}, (\partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu)] \to \text{OK}$$

$$(d+d') + gf^{abc}t^a A_\nu^b\left(f^{cb'c'} w_{b'}A_\nu^{c'}\right)$$

$$+ gf^{abc}t^a A_\nu^c\left(f^{bb'c'} w_{b'}A_\mu^{c'}\right) =$$

$$= -ig\;[t^b t^c]\, A_\mu^b\left(f^{cb'c'} w_{b'}A_\nu^{c'}\right)$$

$$-ig\;[t^b t^c]\, A_\nu^c\left(f^{bb'c'} w_{b'}A_\mu^{c'}\right) =$$

$$= -ig\left(\hat{A}_\mu\, t^c f^{cb'c'} w_{b'}A_\nu^{c'} - t^c f^{cb'c'} w_{b'}A_\nu^{c'}\,\hat{A}_\mu\right)$$

$$\left(\ldots t^b f^{bb'c'} \ldots \hat{A} \ldots - \hat{A} \ldots t^b f^{bb'c'} w_{b'} A^{c'}\right) -$$

$$-ig\left(t^{b} f\ w_{b'} A_\mu^{c'} A_\nu - A_\nu t^{?} f\ w_{b} A_\mu\right) -$$

$$= -ig\left(\hat{A}_\mu (-i)[t^{b'}, t^{c'}] w_{b'} A_\nu^{c'} - (-i)[t^{b'}, t^{c'}] w_{b'} A_\nu^{c'} \hat{A}_\mu\right]$$

$$-ig\left((-i)[t^{b'}, t^{c'}] w_{b'} A_\mu^{c'} \hat{A}_\nu - (-i)\hat{A}_\nu [t^{b'}, t^{c'}] w^{b'} A_\mu^{c'}\right]$$

$$= -g\left[\cancel{\hat{A}_\mu \hat{w} \hat{A}_\nu} - \hat{A}_\mu \hat{A}_\nu \hat{w} - \hat{w} \hat{A}_\nu \hat{A}_\mu + \cancel{\hat{A}_\nu \hat{w} \hat{A}_\mu}\right]$$

$$-g\left[\hat{w} \hat{A}_\mu \hat{A}_\nu - \cancel{\hat{A}_\mu \hat{w} \hat{A}_\nu} - \cancel{\hat{A}_\nu \hat{w} \hat{A}_\mu} + \hat{A}_\nu \hat{A}_\mu \hat{w}\right]$$

$$= -g\left[\hat{w} \hat{A}_\mu \hat{A}_\nu - \hat{w} \hat{A}_\nu \hat{A}_\mu - (\hat{A}_\mu \hat{A}_\nu - \hat{A}_\nu \hat{A}_\mu)\hat{w}\right] =$$

$$= -g\left[\hat{w} [\hat{A}_\mu, \hat{A}_\nu] - [\hat{A}_\mu, \hat{A}_\nu] \hat{w}\right] =$$

$$= -ig \hat{w} f^{abc} A_\mu^b A_\nu^c t^a + ig f^{abc} A_\mu^b A_\nu^c t^a \hat{w}$$