Lecture14

Monday, November 20, 2017 2:07 PM

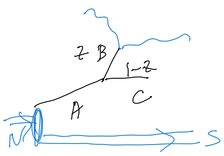
QDD Evolution Equation II

We obtained

$$\tilde{f}(x) = \int_0^1 dy \int_0^1 dz\, \delta(zy-x) f(y) \left[\delta(z-1) + dP_{BA}(z)\right]$$

where

$$dP_{BA}(z)\,dz = \frac{1}{8\pi^2}\frac{z(1-z)}{z}\sum_{spin}\frac{|V_{A\to B+C}|^2}{p_\perp^2}\, dz\, d\ln p_\perp^2 \qquad Eq(1)$$



Symmetry

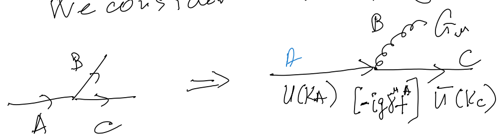
$$dP_{BA}(z) \equiv dP_{CA}(1-z)$$

rename $z \rightleftarrows 1-z$

$$dP_{CA}(z) = dP_{BA}(1-z)$$

⇒ Quark Gluon vertex

We consider now specific case



$$-iM = \bar{u}(k_c)_a\left[-ig\gamma_\mu t^A_{ab} G^A_\mu\, u_b(k_A)\right]$$

$$|M|^2 = \sum_{A,a,b} \bar{u}_b(k_A) G^{A}_{\nu} t^{A}_{ba}\, g\, u(k_c)_a \bar{u}(k_c)_a \times g\gamma_\mu t^A_{ab} G^A_\mu u_b(k_A)$$

→ Colors are fixed in the amplitude level

$$\overline{|M|}^2 = \frac{1}{spin\ pol\ color}|M|^2 = \frac{1}{2\times N}\sum_{AA'} g^2\, tr(\not k_B \gamma_\nu \not k_c \gamma_\mu)\, tr\, t^A t^{A'}$$

$$\boxed{tr\, t^A t^{A'} = \frac{1}{2}\delta^{AA'}} =$$

$$C_2(R) = \frac{N^2-1}{2N}$$

$$= \frac{1}{2}\left(\frac{1}{N}\sum_{AA'}\frac{1}{2}\delta^{AA'}\right) g^2\, tr(\not k_c\gamma_\mu \not k_A \gamma_\nu)\sum G^{*}_{\mu} G_{\nu}$$

$$\sum_{spins}|V_{q\to Gq}|^2 = \frac{g^2}{2}C_2(R)\, Tr(\not k_c \gamma_\mu \not k_A \gamma_\nu)\sum_{pol} G^{*A}_{\mu} G^{A}_{\nu} = //$$

⇒ We consider physical transverse gluons:

$$\sum_{pol} G^{*A}_{\mu} G^{A}_{\nu} = \delta^{ij} - \frac{k_B^i k_B^j}{k_B^2} \qquad i,j = 1,2,3$$

$$// = g^2\frac{4}{2}C_2(R)\left[k_c^\mu k_A^\nu + k_c^\nu k_A^\mu - g^{\mu\nu} k_c k_A\right] \times \left[\delta^{ij} - \frac{k_B^i k_B^j}{k_B^2}\right] =$$

$$= g^2 2\, C_2(R)\left[2\vec{k}_c\vec{k}_A - \frac{2(\vec{k}_c\vec{k}_B)(\vec{k}_A\vec{k}_B)}{k_B^2} + 3(k_c k_A) - \frac{\vec{k}_B^2}{k_B^2}(k_c k_a)\right] =$$

$$= g^2 2\, C_2(R)\left[2\vec{k}_c\vec{k}_A - \frac{2(\vec{k}_c\vec{k}_B)(\vec{k}_a\vec{k}_B)}{k_B^2} + 2(k_c k_A)\right] \quad (*)$$

Definition of momenta

$k_A = (p, p, 0)$

$k_B = \left(zp + \frac{p_\perp^2}{2zp}, zp, p_\perp\right)$

$k_C = \left((1-z)p + \frac{p_\perp^2}{2(1-z)p}, (1-z)p - p_\perp\right)$

$\vec{k}_c \cdot \vec{k}_A = (1-z)p^2$

$\vec{k}_c \cdot \vec{k}_B = z(1-z)p^2 - p_\perp^2$

$\vec{k}_A \cdot \vec{k}_B = zp^2$

$\vec{k}_B^2 = z^2p^2 + p_\perp^2$

$(k_c k_A) = (1-z)p^2 + \frac{p_\perp^2}{2(1-z)} - (1-z)p^2 = \frac{p_\perp^2}{2(1-z)}$

Inserting into (*) one obtains

$$g^2 4\, C_2(R)\left[(1-z)p^2 - \frac{(z(1-z)p^2 - p_\perp^2)zp^2}{z^2p^2 + p_\perp^2} + \frac{p_\perp^2}{2(1-z)}\right]$$

$$= g^2 4\, C_2(R)\left[(1-z)p^2 - \frac{([z(1-z)p^2 - p_\perp^2])z}{z^2 + \frac{p_\perp^2}{p^2}} + \frac{p_\perp^2}{2(1-z)}\right] =$$

$$= g^2 4\, C_2(R)\left[\frac{(1-z)p^2 z^2 + (1-z)p^2\frac{p_\perp^2}{p^2} - z^2(1-z)p^2 + p_\perp^2 z}{z^2 + \frac{p_\perp^2}{p^2}} + \frac{p_\perp^2}{2(1-z)}\right] = \qquad \frac{p_\perp^2}{p^2} \to 0$$

$$= g^2 4\, C_2(R)\left[\frac{\overbrace{(1-z)p_\perp^2 + p_\perp^2 z}^{p_\perp^2}}{z^2} + \frac{p_\perp^2}{2(1-z)}\right]$$

$$= g^2 2\, C_2(R)\left[\frac{2(1-z)p_\perp^2 + z^2 p_\perp^2}{z^2(1-z)}\right] = 2C_2 p_\perp^2\left(\frac{2-2z+z^2}{z^2(1-z)}\right) =$$

$$= \frac{g^2 2\, C_2(R)\, p_\perp^2}{z(1-z)}\left[\frac{1+(1-z)^2}{z}\right]$$

$\Rightarrow$ Inserting in Eq(1)

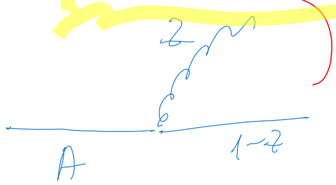
$$dP_{Gq} = \frac{g^2}{8\pi^2}\,\frac{z(1-z)}{z}\,\frac{2\cdot C_2(R)}{z(1-z)}\left[\frac{1+(1-z)^3}{z}\right] d\ln P_\perp^2$$

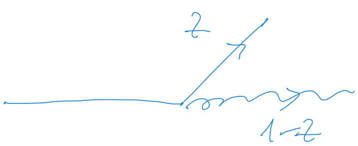
$$= \frac{\alpha}{2\pi}\, C_2(R)\left[\frac{1+(1-z)^2}{z}\right] d\ln P_\perp^2 =$$

$$= \frac{\alpha}{2\pi}\, P_{Gq}(z)\, d\ln P_\perp^2$$

└ splitting function

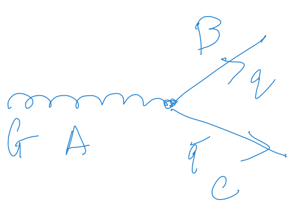
$$P_{Gq}(z) = C_2(R)\left[\frac{1+(1-z)^2}{z}\right]; \quad C_2(R) = \frac{N^2-1}{2N}$$





$$P_{qq}(z) = P_{Gq}(1-z) = C_2(R)\left[\frac{1+z^2}{1-z}\right]$$

⟹ Calculation of Gluon → $q\bar{q}$ splitting



$$-iM_{G\to q\bar{q}} = \bar{u}_a(k_B)\, \bar{v}_b(k_C)\left[-ig\,\gamma_\mu\, t^A_{ab}\, G^A(k_A)\right] =$$

$$= -\bar u_a(k_B)\left[-ig\gamma_\mu t_{ab} G^{\prime}(k_A)\right] u_b(k_C)$$

$$|\overline{M}|^2 = \frac{1}{2}\frac{1}{N^2-1}|M_{G\to q\bar q}|^2 = \frac{1}{2}\frac{1}{N^2-1} g^2 \sum_{\substack{a,b,A \\ \text{spins}}} \bar u_b(k_C)\gamma^\mu u_a(k_B)\bar u(k_B)\gamma^\nu u_b(k_C)\cdot$$

(gluons spin)

$$t^{A'}_{ba}\, t^{A}_{ab}\; G^{+A}_{\mu}\, G^{A'}_{\nu} =$$

$$= \frac{g^2}{N^2-1}\sum_{AA'} Tr(\not k_B\gamma_\mu \not k_C\gamma_\nu)\, tr\, t^{AA'} \times \frac{1}{2}\sum_{spic} G^{+A}_{\mu} G^{A'}_{\nu}$$

$tr\, t^{AA'} = \frac{1}{2}\delta_{AA'}$, $\sum \delta_{AA} = N^2-1$

$$= \frac{g^2}{2} Tr(\not k_C\gamma_\mu \not k_B\gamma_\nu)\frac{1}{2}\sum G^{+A}_{\mu} G^{A}_{\nu} = ||$$

using $\sum_{pol} G^{+A}_{\mu} G^{A}_{\nu} = \left(\delta^{ij} - \frac{k_A^i k_A^j}{\vec{k}_A^2}\right)$

$$|| = \frac{g^2}{4}\, 4\left(k_C^\mu k_B^\nu + k_C^\nu k_B^\mu - g^{\mu\nu}(k_B k_C)\right)\left(\delta^{ij} - \frac{k_A^i k_A^j}{\vec{k}_A^2}\right) =$$

$$= g^2\left(2\vec{k}_C\vec{k}_B - 2\frac{(\vec{k}_C\vec{k}_A)(\vec{k}_B\vec{k}_A)}{\vec{k}_A^2} + 3(k_B k_C) - \frac{\vec{k}_A^2 (k_B k_C)}{\vec{k}_A^2}\right)$$

$$= \frac{g^2}{\;}\, 2\left(\vec{k}_C\vec{k}_B - \frac{(\vec{k}_C\vec{k}_A)(\vec{k}_B\vec{k}_A)}{\vec{k}_A^2} + (k_B k_C)\right]$$

(*)

Definition of Momentum

$$k_A = (P,\ P,\ 0)$$

$$k_B = \left(\frac{zP + P_\perp^2}{2zP},\ zP,\ P_\perp\right)$$

$$k_C = \left(\frac{(1-z)P + P_\perp^2}{2(1-z)P},\ (1-z)P,\ -P_\perp\right)$$

$$\vec{k}_C\cdot\vec{k}_B = z(1-z)p^2 - p_\perp^2$$

$$\vec{k}_B\cdot\vec{k}_A = z p^2$$

$$\vec{k}_A^{\,2} = p^2$$

$$\vec{k}_C\cdot\vec{k}_A = (1-z)p^2$$

$$(k_B k_C) = z(1-z)p^2 + p_\perp^2\frac{(1-z)p}{2zp} + \frac{zp}{2(1-z)p} - z(1-z)p^2 + p_\perp^2$$

$$= p_\perp^2\left(\frac{1-z}{2z} + \frac{z}{2(1-z)}\right) + p_\perp^2 =$$

$$= p_\perp^2\left(\frac{(1-z)^2 + z^2 + 2z(1-z)}{2z(1-z)}\right) =$$

$$= p_\perp^2\left(\frac{(1-z+z)^2}{2z(1-z)}\right) = p_\perp^2\frac{1}{2z(1-z)}$$

Eq (*) becomes

$$|| = 2g^2\left(z(1-z)p^2 - p_\perp^2 - \frac{(1-z)p^2\, z p^2}{p^2} + \frac{p_\perp^2}{2z(1-z)}\right) =$$

$$= 2g^2\left(\frac{p_\perp^2}{2z(1-z)} - p_\perp^2\right) = 2g p_\perp^2\left(\frac{1-2z(1-z)}{2z(1-z)}\right) =$$

$$z^2 + (1-z)^2$$

$$= 2g^2 p_\perp^2\left(\frac{1-2z+2z^2}{2z(1-z)}\right) = \frac{2g^2 p_\perp^2}{z(1-z)}\,\frac{1}{2}\left(z^2+(1-z)^2\right)$$

Using Eq (1)

$$dP_{qG} = \frac{1}{8\pi^2}\,\frac{z(1-z)}{z}\,\frac{2g^2 p_\perp^2}{z(1-z)}\,\frac{1}{p_\perp^2}\,\frac{1}{2}\left(z^2+(1-z)^2\right) dp_\perp^2 =$$

$$= \frac{\alpha}{2\pi} \frac{1}{2} \left(z^2 + (1-z)^2\right) d \ln p_\perp^2 =$$

$$= \frac{\alpha}{2\pi} P_{qG}(z) \; d \ln p_\perp^2$$

$$P_{qG}(z) = \frac{1}{2} \left(z^2 + (1-z)^2\right)$$

$$P_{qG}(z) = P_{\bar{q}G}(1-z)$$