Lecture2

Wednesday, August 30, 2017 2:46 PM

How Interaction Happens

⇒ Amplitude of the Scattering

$$S_{m,e} = \Phi_m^{\dagger} \hat{S} \Phi_e$$

⇒ Consider lowest order in P.T.

$$S^{(1)} = -ie \int \bar{\psi} \not{A} \psi \, d^4x,$$

Produced $e \ \gamma$

— using

$$\psi = \sum_{p_1, \lambda_1} \frac{1}{\sqrt{2\varepsilon_{p_1}}} a_{p_1,\lambda_1} U(p_1,\lambda_1) e^{-ip_1 x}$$

$$\bar{\psi} = \sum_{p_2, \lambda_2} \frac{1}{\sqrt{2\varepsilon_{p_2}}} a^{\dagger}_{p_2\lambda_2} \overline{U(p_2,\lambda_2)} e^{ip_2 x}$$

$$A^{\dagger\mu} = \sum_{k, \nu} \frac{1}{\sqrt{2\omega_k}} e_{\mu}(k,\nu) \, C^{\dagger}_{k,\nu} \, e^{ikx}$$

$\varepsilon_k^{\dagger\mu}$ $U_{p_1\lambda_1}$

$$S^{(1)} = -ie \sum_{\substack{p_1\lambda_1 \\ p_2\lambda_2 \\ k\,\nu}} \frac{\bar{u}(p_2)_{\lambda_2}\, C_{k,\nu}\; u(p_1,\lambda_1)_\nu}{\sqrt{2\varepsilon_{p_2}}\sqrt{2\varepsilon_{p_1}}\sqrt{2\omega_k}}$$

$$e^{i(\tilde{k}+\tilde{p}_2-p_1)}\, d^4x \; -ie\gamma^\mu$$

$$= \frac{(2\pi)^4\,\delta^4(k+p_2-p_1)}{\sqrt{2\varepsilon_{p_2}}\sqrt{2\varepsilon_{p_1}}\sqrt{2\omega_{k}}}\; a^+_{p_2\lambda_2}\, C^+_{k,\lambda}\, a_{p_1\lambda_1}$$

$$S_{e\to e\gamma} = \langle \phi_{\substack{1,1\\ e\,\gamma}} |\; \hat{S}^{(1)}\, | \phi_{1,0} \rangle$$

$$|\phi_{1\,0}\rangle = a^+_{p_i\lambda_i}|0\rangle$$

$$|\phi_{1\,1}\rangle = a^+_{p_f\lambda_f}\, C^+_{k_f,\nu_f}|0\rangle$$

$$S_{e\to e\gamma} = \langle 0 | a_{p_f\lambda_f}\, C_{k_f\nu_f} \sum_{\substack{p_1\lambda_1 \\ p_2\lambda_2 \\ k\,\nu}} a^+_{p_2\lambda_2}\, C^+_{k\,\nu}$$

$$\bar{u}(p_2\ldots)\ \ldots e^{ikx^\mu}\ \ldots u(p_1)\, x(p_1\lambda_1 \ldots$$

$$U(p_2,\lambda_2)\,\varepsilon^{\nu}_{k}(\nu)\,[\ldots$$

$$\times\, a^\dagger_{p_i\lambda_i}|0\rangle$$

$\Rightarrow$ using

$$[c_{k\nu},\, c^\dagger_{k'\nu'}] = \delta_{kk'}\,\delta_{\nu\nu'}$$

$$\{a_{p\lambda},\, a^\dagger_{p'\lambda'}\} = \delta_{pp'}\,\delta_{\lambda\lambda'}$$

$$a_{p_f\lambda_f}\, a^\dagger_{p_2\lambda_2} = \delta_{p_f p_2}\,\delta_{\lambda_f\lambda_2} - a^\dagger_{p_2\lambda_2}\, a_{p_f\lambda_f}$$

$$c_{k_f\nu_f}\, c^\dagger_{k\nu} = \delta_{k_f k}\,\delta_{\nu_f\nu} + c^\dagger_{k\nu}\, c_{k_f\nu_f}$$

$$a_{p_1\lambda_1}\, a^\dagger_{p_i\lambda_i} = \delta_{p_i p_1}\,\delta_{\lambda_i\lambda_1} - a^\dagger_{p_i\lambda_i}\, a_{p_1\lambda_1}$$

$\Rightarrow$ using the fact that

$$a_{p\lambda}|0\rangle = 0$$
$$\langle 0|a^\dagger_{p\lambda} = 0$$
$$c_{k\nu}|0\rangle = 0$$
$$\langle 0|c^\dagger_{k\nu} = 0$$

One obtains

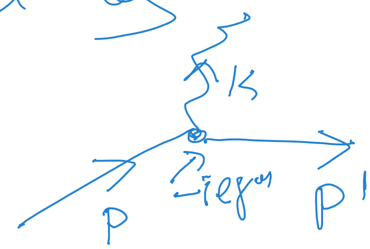
$$S_{e\to e\gamma} = \frac{(2\pi)^4\,\delta^4(P_f+k_f-k_i)}{\ldots}\;\overbrace{\bar U_{p_f\lambda_f}\,[-ie\gamma^\mu]\,U_{p_i\lambda_i}\,\epsilon^{\mu}(k\nu)}^{-i\mathcal{M}}$$

$$-i\mathcal{M} = \varepsilon^{*}_{\mu f}\,\bar{u}_{p_f\lambda_f}\left[-ie\gamma^{\mu}\right]u_{p_i\lambda_i} = \bar{u}_{p_f\lambda_f}\left[-ie\gamma^{\mu}\,\varepsilon^{\mu}_{(\mu)}\right]u_{p_i\lambda_i}$$

- Vertex of interaction $[-ie\gamma^{\mu}]$

⇒ if we go to the Lagrange Density it can be obtained by

$$\frac{\delta \mathcal{L}}{\delta\bar{\Psi}\,\delta\Psi\,\delta A^{\mu}} = -ie\gamma^{\mu}$$



- propagators

⇒ Consider the process with initial $e\gamma$ and another at the end $e\gamma$

⇒ One need

$$[-ie]^2\, T \int \bar{\Psi}(x_2)\,\not{A}(x_2)\,\Psi(x_2)\,d^4x_2$$

$$\int \bar{\Psi}(x_1)\cancel{A}(x_1)\,\Psi(x_1)\,d^4x_1 =$$

$t_2 > t_1$

$$= [-ie^2]\int \bar{\Psi}(x_2)\cancel{A}(x_2)\overbrace{\Psi(x_2)\bar{\Psi}(x_1)}^{\theta(t_2-t_1)}\times \cancel{A}(x_1)\,\Psi(x_1)\,d^4x_1\,d^4x_2$$

$\Rightarrow$ Remember $K(\overset{x_2}{r_2 t_2}, \overset{x_1}{r_1 t_1}) = \sum_n \psi_n(r_2 t_2)\,\psi_n^*(r_1 t_1)$

$$G(r_2 r_1) = \theta(t_2 - t_1)\,K(r_2 t_2, r_1 t_1)$$

$$\left[i\frac{\partial}{\partial t_2} - H(r_2, t_2)\right]G(r_2 r_1) = i\,\delta^3(r_2 - r_1)\,\delta(t_2 - t_1)$$

$$G(r,t) = \int \frac{d^4p}{(2\pi)^4}\,G_0(p, E)\,e^{i\vec{p}\vec{r} - iEt}$$

$$G_0(p) = \frac{i}{\left(p_0 - \frac{p^2}{2m} + i\varepsilon\right)} \qquad \boxed{p_0\psi = \frac{p^2}{2m}\psi}$$

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using this

$$G_e(x_2,x_1) = \theta(t_2-t_1)\,\psi(x_2)\,\bar{\psi}(x_1)$$

$$(i\gamma^\mu\partial_\mu - m)\,G_0(x_2,x_1) = i\,\delta^4(x_2-x_1)$$

$$G_0(x_2-x_1) = \int G(P)\,e^{-iPX}\,\frac{d^4P}{(2\pi)^4}$$

$$\int (i\gamma_\mu\partial_\mu - m)\,e^{-iPX}\,G(P)\,\frac{d^4P}{(2\pi)^4} =$$

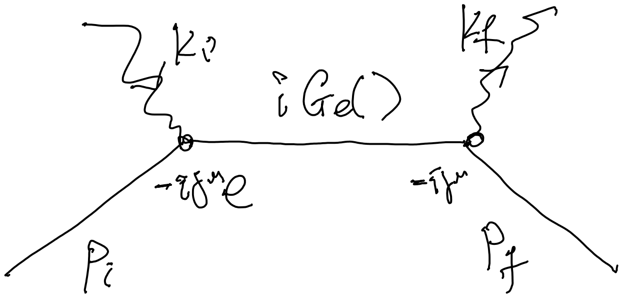
$$= (P_\mu\gamma_\mu - m)\,G(P)\;\delta^4(x) = i\,\delta^4(x)$$

$$\boxed{G_e(P) = \frac{i}{\not{P} - m}}$$

$\Rightarrow$ Feynman Diagram

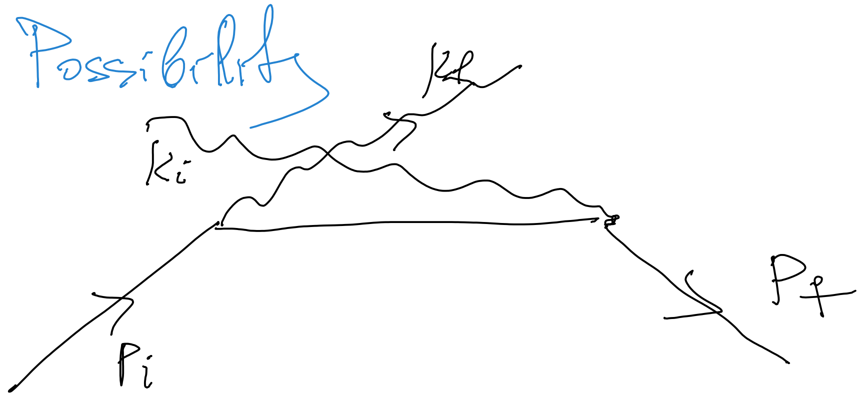
$$S_{e\gamma\to e\gamma} = (2\pi)^4\,\delta^4(K_i+P_i-K_f+P_f)\,[-iM]$$

$e^\mu(k,x) = \varepsilon^\mu(k,\lambda)$



$$-iM_1 = \bar{U}(P_f,\lambda_f)\left[-i\gamma^\sigma e\,\varepsilon^*_\sigma(k_f,\nu_f)\right]\frac{i}{(\cancel{P_i}+\cancel{K_i})-m+i\varepsilon}\left[-ie\gamma^\mu\,\varepsilon_\mu(K_i,\nu_i)\right]U(P_i,\lambda_i)$$

⇒ Show also That there is another Possibility



$$-iM_2 = \bar{U}(P_f,\lambda_f)\left[-ie\gamma^\mu\,\varepsilon_\mu(K_i,\nu_i)\right]\frac{i}{(\cancel{P_i}-\cancel{K_f})-m+i\varepsilon}\left[-ie\gamma^\nu\,\varepsilon^*_\nu(K_f,\nu_f)\right]U(P_i,\lambda_i)$$

$$\boxed{-iM = -iM_1 - iM_2}$$

$$S_{if} = \frac{(2\pi)^4\,\delta^4(K_i+P_i-K_f-P_f)}{\sqrt{K_0^i}\,\sqrt{P_0^i}\,\sqrt{K_0^f}\,\sqrt{P_0^f}}\left[-iM\right]$$

$$W_{if} = |S_{if}|^2$$