$$\hat{S} = T e^{i\int \mathcal{L}' d^4x}$$

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Feynman Rules by Greiner, Schramm, Stein } GSS

⇒ Feynman rules are obtained by varying the corresponding action integral in momentum space

action → $S_{int} = \int L_{int}\, d^4x$ (not a S matrix)

[used as I in the class]

Consider qqG vertex

$$S_{int} = -\int \bar{q}^j_\beta(x)\, t^a \gamma^\mu_{\beta\alpha} G^a_\mu(x)\, q^i_\alpha(x)\, d^4x\, g =$$

$$= -\int \bar{q}^j_\beta(x)\, t^a_{ji}\, \gamma^\mu_{\beta\alpha}\, G^a_\mu(x)\, q^i_\alpha(x)\, d^4x\, g = //$$

For considering momentum space we us (outgoing)

$$q_i(x) = \int q_i(p)\, e^{-ipx} d^4p, \quad G_\nu^a = \int G_\nu^a(p)\, e^{ipx} d^4p$$

$$= -\int \bar{q}_\beta^j(p_1)\, e^{ip_1x} d^4p_1\; t^a_{ji}\, \gamma^\mu_{\beta\alpha}\, G^a_\nu(p_2)\, e^{ip_2x} d^4p_2\; q^i_\alpha(p_3)\, e^{-ip_3x} d^4p_3\, d^4x$$

$$= -\int \bar{q}^j_\beta(p_1)\, t^a_{ji}\, \gamma^\mu_{\beta\alpha}\, G^a_\mu(p_2)\, q^i_\alpha(p_3)\,(2\pi)^4 \delta^4(p_1+p_2-p_3)\, d^4p_1\, d^4p_2\, d^4p_3$$

Vertex is obtained by calculating

$$ i \frac{\delta^3}{\delta q^e_\gamma(p)\,\delta \bar q^m_\sigma(p')\,\delta G^{b\nu}(k)} \mathcal{L}_{int} = // $$

using $\frac{\delta \psi(p)^i_\gamma}{\delta \psi(p')^j_\beta} = \delta^4(p_1-p')\,\delta_{ij}\,\delta_{\gamma\beta}\, g^{\alpha\beta}$ $\quad \prod d^4p \quad (2\pi)^4\delta(p_1+p_2-p_3)$

$$ // = -i\frac{\delta^3}{\delta q^e_\gamma(p)\,\delta\bar q^m_\sigma(p')\,\delta G^{b\nu}(k)} \int \bar q^j_\rho(p_1)\, t^a_{ji}\,\gamma^\mu_{\rho\alpha}\, G^a_\mu(p_2)\, q^i_\alpha(p_3)\, g $$

$$ // = -i\int g\,\delta^{mj} g^{\beta\sigma}\,\delta^4(p_1-p')\, t^a_{ji}\,\gamma^\mu_{\rho\alpha}\; \delta^{ba} g_{\mu\nu}\,\delta(p_2-k)\; \delta^{ie} g^{\gamma\alpha}\,\delta(p_3-p)\; (2\pi)^4\delta(p_1+p_2-p_3)\, d^4p_1\, d^4p_2\, d^4p_3 $$

$$ = -i\,g\,t^b_{me}\,\gamma^\nu_{\sigma\gamma}\,(2\pi)^4\,\delta(p'+k-p) $$

$$ = -i\,g\,t^b\gamma^\nu\,(2\pi)^4\,\delta^4(p'+k-p) $$

Quadro

$\Rightarrow$ Gluon (Triple) Vertex

$$\mathcal{L}_G = -\frac{1}{4}\sum_{\mu,\nu=0}^{3}\sum_{a=1}^{N_c^2-1} F_{\mu\nu}^{a} F^{\mu\nu a} = //$$

$$F_a^{\mu\nu} = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g f^{abc} G_\mu^b G_\nu^c$$

$$// = -\frac{1}{4}\Big\{ \left(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g f^{abc} G_\mu^b G_\nu^c\right)$$

$$\left(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g f^{amn} G_\mu^m G_\nu^n\right)$$

$\Rightarrow$ interaction part where there is a g: it will be first power of g and g^2

$$+\frac{1}{4}\left(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g f^{abc} G_\mu^b G_\nu^c\right) g f^{amn} G_\mu^m G_\nu^n$$

$$+\frac{1}{4} g f^{abc} G_\mu^b G_\nu^c \left(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g f^{amn} G_\mu^m G_\nu^n\right)$$

g term:

$$+\frac{1}{4}(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a)\, g\, f^{abc} G_\mu^b G_\nu^c$$

$$+\frac{1}{4}\, g f^{abc} G_\mu^b G_\nu^c (\partial_\mu G_\nu^a - \partial_\nu G_\mu^a)$$

$$= \frac{g}{2}(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a) f^{abc} G_\mu^b G_\nu^c$$

$$= \frac{g}{2}\partial_\mu G_\nu^a f^{abc} G_\mu^b G_\nu^c - \frac{g}{2}\partial_\nu G_\mu^a f^{abc} G_\mu^b G_\nu^c$$

$$-\frac{g}{2}\partial_\mu G_\nu^a f^{abc} G_\nu^b G_\mu^c$$

($\mu \rightleftarrows \nu$) $f^{abc} = -f^{acb}$

$b \rightleftarrows c$

$$\frac{g}{2}\partial_\mu G_\nu^a f^{abc} G_\mu^b G_\nu^c$$

$$= g\, \partial_\mu G_\nu^a f^{abc} G_\mu^b G_\nu^c$$

$$L^{3g}_{Int} = g\int \partial_\mu G^a_\nu(x) f^{abc} G^b_\mu(x) G^c_\nu(x)\, d^4x$$

$$G^a_\nu(x) = \int G^a_\nu(P_1)\, e^{-iP_1 x}\, d^4P$$

In Momentum Space

$$L^{3g}_{Int} = g\int \partial_\mu\; G^a_\nu(P_1) e^{-iPx} d^4P_1 f^{abc}\; G^b_\mu(P_2) e^{-iP_2x} d^4P_2\; G^c_\nu(P_3) e^{-iP_3x} d^4P_3\; d^4x$$

$$= g\int -iP_1^\mu\; G^a_\nu(P_1)\, f^{abc}\; G^b_\mu(P_2)\, G^c_\nu(P_3)\; (2\pi)^4\delta(P_1+P_2+P_3)\; d^4P_1 d^4P_2 d^4P_3$$

$$= -i\, g P_1^\mu\, G^a_\nu(P_1)\, f^{abc}\, G^b_\mu(P_2)\, G^c_\nu(P_3)\; (2\pi)^4\delta(\Sigma P)$$

$$i\frac{\delta^3}{\delta G^r_\rho(K_1)\,\delta G^s_\sigma(K_2)\,\delta G^t_\tau(K_3)} -i\, g P_1^\mu G^a_\nu(P_1) f^{abc} G^b_\mu(P_2) G^c_\nu(P_3) =$$

$$= g\frac{\delta^2}{G^r_\rho(K_1)\,\delta G^s_\sigma(K_2)}\left[\begin{array}{l} P_1^\mu\, \delta(P_1-K_3)\, g^{\tau\nu}\, \delta^{at} f^{abc} G^b_\mu(P_2) G^c_\nu(P_3) \\ P_1^\mu G^a_\nu(P_1) f^{abc}\, \delta(P_2-K_3)\, g^{\mu\tau}\, \delta^{bt}\, G^c_\nu(P_3) \\ P_1^\mu G^a_\nu\, f^{abc}\, \delta(P_3-K_3)\, G^b_\mu(P_2)\, \delta(P_3-K_3)\; g^{\tau\nu}\, \delta^{tc} \end{array}\right.$$

$\delta(P_1+P_2+P_3)$

$$= g\frac{\delta^2}{G_\rho^r(K_1)\,G_\sigma^s(K_2)}\Big[K_3^\mu f^{tbc} G_\mu^b(P_2)\,G_\tau^c(P_3)\,\delta(K_3+P_2+P_3)$$

$$P_1^\tau G_\nu^a(P_1)\, f^{atc}\, G_\nu^c(P_3)\,\delta(P_1+K_3+P_3)$$

$$P_1^\mu G_\tau^a(P_1)\, f^{abt}\, G_\mu^b(P_2)\,\delta(P_1+P_2+K_3)\Big]$$

$$= g\frac{\delta}{G_\rho^r(K_1)}\Big[K_3^\mu f^{tbc}\,\delta(P_2-K_2)\, g_{\mu\sigma}\,\delta^{bs}\, G_\tau^c(P_3)\,\delta(K_3+P_2+P_3)$$

$$+ K_3^\mu f^{tbc}\, G_\mu^b(P_2)\,\delta(P_3-K_2)\, g^{\tau\sigma}\,\delta^{sc}\,\delta(K_3+P_2+P_3)$$

$$+ P_1^\tau\,\delta(P_1-K_2)\, g^{\nu\sigma}\,\delta^{sa}\, f^{atc}\, G_\nu^c(P_3)\,\delta(P_1+K_3+P_3)$$

$$+ P_1^\tau G_\nu^a(P_1)\, f^{atc}\,\delta(K_2-P_3)\, g_{\nu\sigma}\,\delta^{sc}\,\delta(P_1+K_3+P_3)$$

$$+ P_1^\mu\,\delta(P_1-K_2)\, g_{\tau\sigma}\,\delta^{sa}\, f^{abt}\, G_\mu^b(P_2)\,\delta(P_1+P_2+K_3)$$

$$P_1^\mu G_\tau^a(P_1)\, f^{abt}\,\delta(P_2-K_2)\, g^{\mu\sigma}\,\delta^{sb}\,\delta(P_1+P_2+K_3)$$

$$= g\frac{\delta}{G_\rho^r(K_1)}\Big[K_3^\sigma f^{tsc}\, G_\tau^c(P_3)\,\delta(K_2+K_2+P_3)$$

$$K_3^\mu f^{tbs}\, G_\mu^b(P_2)\, g^{\tau\sigma}\,\delta(K_3+P_2+K_2)$$

$$K_2^\tau f^{stc}\, G_\sigma^c(P_3)\,\delta(K_2+K_3+P_3)$$

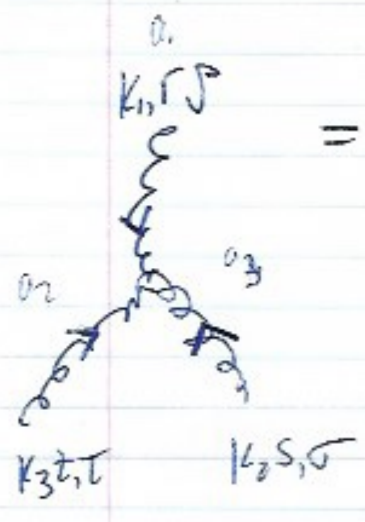
$$P_1^\tau G_\sigma^a(P_1)\, f^{ats}\,\delta(P_1+K_2+K_2)$$

$$K_2^\mu g_{\tau\sigma}\, f^{sbt}\, G_\mu^b(P_2)\,\delta(K_2+P_2+K_3)$$

$$P_1^\sigma G_\tau^a(P_1)\, f^{ast}\,\delta(P_1+K_2+K_2)\Big]$$

$$= g \left[\begin{array}{ll} k_3^{\sigma} f^{tsr} g^{\tau\rho} \delta(k_2+k_3+k_1) & 1 \\ k_3^{\rho} f^{trs} g^{\tau\sigma} \delta(k_3+k_1+k_2) & 2 \\ k_2^{\tau} f^{str} g^{\rho\sigma} \delta(k_2+k_3+k_1) & 3 \\ k_1^{\tau} g^{\sigma\rho} f^{rts} \delta(k_1+k_3+k_2) & 3 \\ k_2^{\rho} g^{\tau\sigma} f^{srt} \delta(k_2+k_1+k_3) & 2 \\ k_1^{\sigma} g^{\tau\rho} f^{rst} \delta(k_1+k_2+k_3) & 1 \end{array} \right)$$

$$g \left[\begin{array}{l} \left(k_3^{\sigma} f^{tsr} + k_1^{\sigma} f^{rst}\right) g^{\tau\rho} \\ \left(k_3^{\rho} f^{trs} + k_2^{\rho} f^{srt}\right) g^{\tau\sigma} \\ \left(k_2^{\tau} f^{str} + k_1^{\tau} f^{rts}\right) g^{\sigma\rho} \end{array} \right]$$



$$= g f^{rst} \left[\begin{array}{l} \left(k_1^{\sigma} - k_3^{\sigma}\right) g^{\tau\rho} \\ \left(k_3^{\rho} - k_2^{\rho}\right) g^{\tau\sigma} \quad (+) \\ \left(k_2^{\tau} - k_1^{\tau}\right) g^{\sigma\rho} \end{array} \right.$$

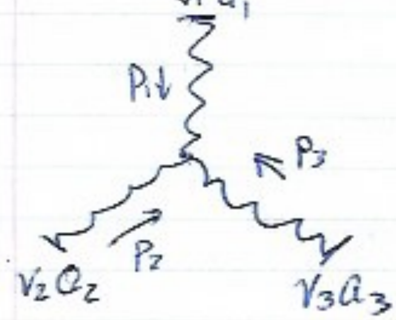
For the Notation of Handbook

we have

$k_1 \to p_1 \quad \vartheta \to \nu_1 \quad r \to a_1$

$k_2 \to p_3 \quad \sigma \to \nu_3 \quad s \to a_3$

$k_3 \to p_2 \quad \tau \to \nu_2 \quad t \to a_2$



Vertex becomes

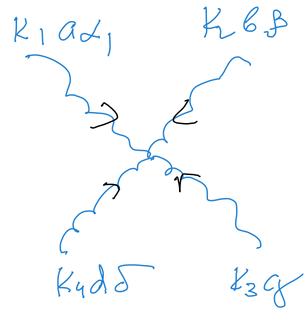
$$= g f^{a_1 a_3 a_2}\left[(p_1 - p_2)^{\nu_3} g^{\nu_2 \nu_1} + (p_2 - p_3)^{\nu_1} g^{\nu_2 \nu_3} + (p_3 - p_1)^{\nu_2} g^{\nu_3 \nu_1} \right] =$$

$$= -g f^{a_1 a_2 a_3}\left[g^{\nu_1 \nu_2} (p_1 - p_2)^{\nu_3} + g^{\nu_2 \nu_3} (p_2 - p_3)^{\nu_1} + g^{\nu_3 \nu_1} (p_3 - p_1)^{\nu_2} \right]$$

Lecture8_b

Wednesday, October 18, 2017 2:36 PM

⇒ Four-Gluon Vertex



$$F = -\frac{1}{4}\sum_{\mu,\nu=0}^{3}\sum_{a=1}^{N^2-1} F^a_{\mu\nu} F^a_{\mu\nu}$$

$$F^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g f^{abc} G^b_\mu G^c_\nu$$

– For 4-Gluon Interaction Look for g^2 term

$$\mathcal{L}^{4g} = -\frac{1}{4} g^2 \sum_{\mu,\nu}\sum_{a=1}^{N^2-1} f^{efg} G^e_\mu G^f_\nu f^{rsg} G^r_\mu G^s_\nu =$$

$$= -\frac{g^2}{4} f^{efg} f^{rsg} G^e_\mu G^f_\nu G^{r\mu} G^{s\nu}$$

$$I^{4g}_{int} = -\frac{g^2}{4} e^{efg} f^{rsg} \int G^e_\mu(x)\, G^f_\nu(x)\, G^{r\mu}(x)\, G^{s\nu}(x)\, d^4x$$

$$G^e_\mu(x) = \int G^e_\mu(p_1)\, e^{-ip_1 x}\, d^4 p_1$$

$$I^{4g}_{int} = -\frac{g^2}{4} f^{efg} e^{rsg} \int G^e_\mu(p_1) G^f_\nu(p_2)\, G^{r\mu}(p_3) G^{s\nu}(p_4)\, \delta^4(p_1+p_2+p_3+p_4)\, d^4p_1\, d^4p_2\, d^4p_3\, d^4p_4$$

Vertex
$$i\,\frac{\delta I^{4g}_{int}}{\delta G^{a_1}_{\alpha_1}(k_1)\, G^{a_2}_{\alpha_2}(k_2)\, G^{a_3}_{\alpha_3}(k_3)\, G^{a_4}_{\alpha_4}(k_4)}$$

$$4\; e^{efg}\; f^{rsg}\; g^{\mu\alpha_i}\, g^{\nu\alpha_j}\, g^{\mu\alpha_k}\, g^{\nu\alpha_m}$$

$$= -\frac{g^2}{4} \sum_{i,j,k,m=1} f \; f \quad g \; g \; g \; g \quad \delta^{e a_i} \delta^{f a_j} \delta^{r a_k} \delta^{s a_m}$$

$$= -\frac{g^2}{4} \sum_{i,j,k,m=1}^{4} f^{a_i a_j g} f^{a_k a_m g} g^{\alpha_i \alpha_k} g^{\alpha_j \alpha_m} =$$

$$= -\frac{g^2}{4} \sum_{i,j,k,m=1}^{4} \Big\{ f^{a_1 a_2 g} f^{a_3 a_4 g} g^{\alpha_1 \alpha_3} g^{\alpha_2 \alpha_4} \quad 1$$

$$f^{a_1 a_2 g} f^{a_4 a_3 g} g^{\alpha_1 \alpha_4} g^{\alpha_2 \alpha_3} \quad 2$$

$$f^{a_1 a_3 g} f^{a_2 a_4 g} g^{\alpha_1 \alpha_2} g^{\alpha_3 \alpha_4} \quad 3$$

$$(*) \; f^{a_1 a_3 g} f^{a_4 a_2 g} g^{\alpha_1 \alpha_4} g^{\alpha_3 \alpha_2} \quad 4$$

(F)

$$f^{a_1 a_4 g} f^{a_2 a_3 g} g^{\alpha_1 \alpha_2} g^{\alpha_4 \alpha_3} \quad 5$$

$$f^{a_1 a_4} f^{a_3 a_2 g} g^{\alpha_1 \alpha_3} g^{\alpha_4 \alpha_2} \quad 6$$

$\Rightarrow$ All other terms are identical to above

$a_1 \leftrightarrow a_3$

$$(*) \; f^{a_3 a_1 g} f^{a_2 a_4 g} g^{\alpha_3 \alpha_2} g^{\alpha_1 \alpha_4}$$

$\Rightarrow$ Therefore we get the above expression times $\times 4$

$$(2) \; -f^{a_1 a_2} f^{a_3 a_4}$$

replacing above

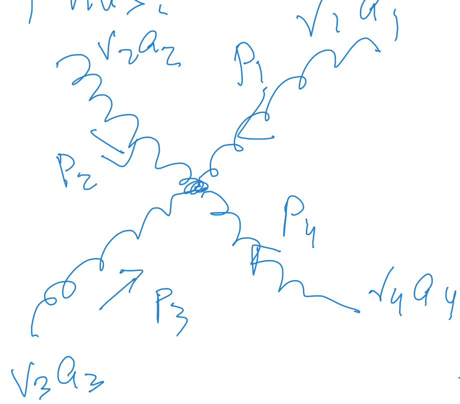
$$f^{a_1 a_3} f^{a_4 a_2}$$

(5) $- f \quad f$

(6) $- f^{a_1 a_4} f^{a_2 a_3}$)

$$\text{Vertex} = -ig^2 \Big[f^{a_1 a_2 g} f^{a_3 a_4 g} \left[g^{\alpha_1 \alpha_3} g^{\alpha_2 \alpha_4} - g^{\alpha_1 \alpha_4} g^{\alpha_2 \alpha_3} \right]$$
$$+ f^{a_1 a_3 g} f^{a_4 a_2 g} \left[g^{\alpha_1 \alpha_4} g^{\alpha_3 \alpha_2} - g^{\alpha_1 \alpha_2} g^{\alpha_3 \alpha_4} \right]$$
$$+ f^{a_1 a_4 g} f^{a_2 a_3 g} \left[g^{\alpha_1 \alpha_2} g^{\alpha_4 \alpha_3} - g^{\alpha_1 \alpha_3} g^{\alpha_4 \alpha_2} \right]$$
$$(2\pi)^4 \delta(P_1 + P_2 + P_3 + P_4)$$

Thus:



$$-ig^2 \Big\{ f^{e a_1 a_2} f^{e a_3 a_4} \left[g^{\nu_1 \nu_3} g^{\nu_2 \nu_4} - g^{\nu_1 \nu_4} g^{\nu_2 \nu_3} \right]$$
$$+ f^{e a_1 a_3} f^{e a_4 a_2} \left[g^{\nu_1 \nu_4} g^{\nu_3 \nu_2} - g^{\nu_1 \nu_2} g^{\nu_3 \nu_4} \right]$$
$$+ f^{e a_1 a_4} f^{e a_2 a_3} \left[g^{\nu_1 \nu_2} g^{\nu_4 \nu_3} - g^{\nu_1 \nu_3} g^{\nu_4 \nu_2} \right] \Big\}$$

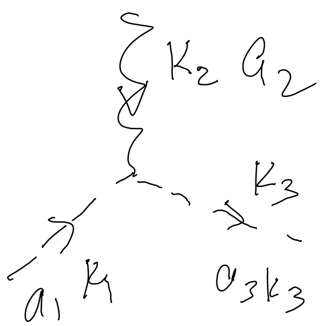
⟹ Ghost field:

$$\mathcal{L}_{int}^{ghost} = -g f^{abc} (\partial_\mu \phi^+)^a A_\mu^b \phi^c$$

$$I_{int}^{ghost} = -g f^{abc} \int \partial_\mu \phi^{+a} A_\mu^b \phi^c \, d^4x =$$

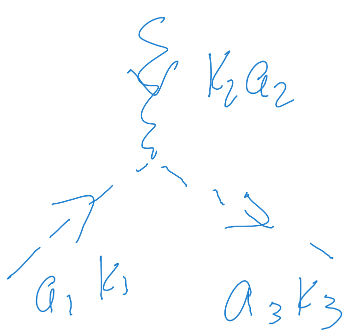
$$= -ig f^{abc} \int P_3^{\mu} \phi^{+a}(P_3)\, G^{b}_{\mu}(P_2)\, \phi^{c}(P_1)\, (2\pi)^4 \delta(P_1+P_2-P_3)\, d^4P_1\, d^4P_2\, d^4P_3 \quad \checkmark$$

$$\frac{i\,\delta I^{ghost}_{int}}{\delta \phi^{+}_{a_3}(K_3)\, \delta G^{a_2}_{\nu}(K_2)\, \delta \phi_{a_1}(K_1)} = g f^{abc} K_3^{\nu}\, \delta^{a a_3}\, \delta^{b a_2}\, \delta^{c a_1}\, (2\pi)^4 \delta(K_1+K_2-K_3)$$



$$= g f^{a_3 a_2 a_1} K_3^{\nu} (2\pi)^4 \delta(K_1+K_2-K_3)$$

$$= -g f^{a_1 a_2 a_3} K_3^{\nu} (2\pi)^4 \delta(K_1+K_2-K_3)$$



$$= -g f^{a_1 a_2 a_3} K_3^{\nu} \checkmark$$

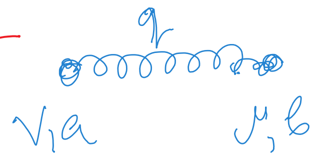
⇒ Propagators

– quark Propagator

$$\frac{i\, \delta_{ij}\, [K^{\mu}\gamma_{\mu} - m]}{\ldots}$$

α, i β, j $K^2 - m + i\varepsilon$

– gluon Propagator



$$\frac{i\delta_{ba}}{q^2+i\varepsilon}\left[-g^{\mu\nu}+(1-\xi)\frac{q_\mu q_\nu}{q^2+i\varepsilon}\right]$$

Covariant Gauge

$$\frac{-i\delta_{ba}}{q^2+i\varepsilon}\left[-g^{\mu\nu}+\frac{k^\mu n^\nu+k^\nu n^\mu}{n\cdot k}-n^2\frac{k^\mu k^\nu}{(n\cdot k)^2}(1-\xi)\right]$$

Axial Gauge

– ghost Propagator

$$\frac{i\delta_{ba}}{K^2+i\varepsilon}$$