MAP 2302 (Differential Equations) — Answers
QUIZ 3, Friday February 2, 2018

Name: **PID:**

1. [4] Transform the Bernoulli differential equation $\frac{dy}{dx}+\cos(x)y = y^3\sin(x)$ into a linear differential equation. But, do not solve the linear equation obtained.

Multiply both sides by y^{-3}:

$(*)$ $y^{-3}\frac{dy}{dx} + \cos(x)\,y^{-2} = \sin(x)$. Set $v = y^{-2}$; $\frac{dv}{dx} = -2y^{-3}\frac{dy}{dx}$

multiply both sides of $(*)$ by -2:

$-2y^{-3}\frac{dy}{dx} - 2\cos(x)\,y^{-2} = -2\sin(x)$, or

$\frac{dv}{dx} - 2\cos(x)v = -2\sin(x)$, which is linear.

2. [6] Solve the initial-value problem: $\begin{cases} (x^2+1)\frac{dy}{dx} + 2xy = x^3 \\ y(0) = 2. \end{cases}$

Divide both sides of D.E by (x^2+1):

$$\frac{dy}{dx} + \frac{2x}{x^2+1}y = \frac{x^3}{x^2+1}$$

General solution is given by

$$y = e^{-\int\frac{2x}{x^2+1}dx}\left[\int e^{\int\frac{2x}{x^2+1}dx}\cdot\frac{x^3}{x^2+1}dx + C\right],\ C = \text{Constant}$$

$$= e^{-\ln(x^2+1)}\left[\int x^3 e^{\ln(x^2+1)}dx + C\right]$$

$$= e^{\ln\frac{1}{x^2+1}}\left[\int \frac{x^3}{\cancel{x^2+1}}\cdot\cancel{(x^2+1)}\,dx + C\right]$$

$$= \frac{1}{x^2+1}\left[\int x^3dx + C\right] = \frac{1}{x^2+1}\left(\frac{x^4}{4}+C\right)$$

Soln of IVP, find C:

$y(0) = C$ and $y(0) = 2$; so $C = 2$

$y = \frac{1}{x^2+1}\left(\frac{x^4}{4}+2\right)$ is the unique solution of IVP.