MAP 2302 (Differential Equations) — Answers
QUIZ 4, Friday February 16, 2018

Name: **PID:**

1. [4] The roots of the auxiliary equation corresponding to a certain 8th-order homogeneous linear differential equation with constant coefficients are -2, -2, 3, 5, $4-5i$, $4+5i$, $4-5i$, $4+5i$. a) Write down the general solution of this equation. b) If a corresponding nonhomogeneous equation has $y_p = \sin(3x) - 5e^x + 6x^3$ as a particular solution, write down the general solution of the nonhomogeneous equation.

a) $y_c = (c_1 + c_2 x)e^{-2x} + c_3 e^{3x} + c_4 e^{5x} + ((c_5 + c_6 x)\cos(5x) + (c_7 + c_8 x)\sin(5x))e^{4x}$
$c_1, \ldots, c_8 = \text{constants}$

b) $y = y_c + y_p$

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2. [4] Use the reduction of order method to solve the differential equation $(x^2+1)y'' - 4xy' = 0$.

Set $v = y'$, then v solves the first-order D.E
$(x^2+1)v' - 4xv = 0$, linear
or $v' - \frac{4x}{x^2+1}v = 0$
So $v(x) = c\,e^{\int \frac{4x}{x^2+1}dx} = c\,e^{2\ln(x^2+1)} = c\,e^{\ln((x^2+1)^2)}$
$= c(x^2+1)^2$, $c = \text{constant}$
solve $y' = c(x^2+1)^2 = c(x^4 + 2x^2 + 1)$
$y = c\left(\frac{x^5}{5} + \frac{2}{3}x^3 + x\right) + d$, $d = \text{constant}$

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3. [2] Solve the differential equation $y^{(iv)} - y = 0$.

Auxiliary equation: $m^4 - 1 = 0$ or $(m^2-1)(m^2+1) = 0$
So $m_1 = 1$, $m_2 = -1$, $m_3 = i$, $m_4 = -i$
$y_c = c_1 e^x + c_2 e^{-x} + c_3 \sin(x) + c_4 \cos(x)$,
$c_1, \ldots, c_4 = \text{constants}$