MAC 2312 (Calculus II) — Answers
QUIZ 5, Friday September 23, 2016

Name: **PID:**

Remember to show all your work; you won't get any credits if you do not show the steps to your answers.

1. [3] Use integration by parts to evaluate the integral:

$$\int_0^{\frac{\pi}{2}} x\cos(2x)\,dx = x\frac{\sin(2x)}{2}\Big]_0^{\pi/2} - \frac{1}{2}\int_0^{\pi/2}\sin(2x)dx = \frac{\pi}{2}\left(\frac{\sin(\pi)}{2} - \frac{\sin 0}{2}\right) + \frac{\cos(2x)}{4}\Big]_0^{\pi/2}$$

$u = x;\ du = dx$

$dv = \cos(2x)dx$

$v = \frac{\sin(2x)}{2}$

$$= \frac{\cos(\pi) - \cos(0)}{4}$$

$$= -\frac{2}{4}$$

$$= -\frac{1}{2}$$

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2. [7] Evaluate each integral:

a) $\int_0^2 \frac{dx}{x^2+4} = \int_0^{\pi/4} \frac{2\sec^2\theta\,d\theta}{4(\tan^2\theta+1)} = \frac{1}{2}\int_0^{\pi/4}\frac{\sec^2\theta}{\sec^2\theta}d\theta = \frac{1}{2}\int_0^{\pi/4} d\theta = \frac{\theta}{2}\Big]_0^{\pi/4} = \frac{\pi}{8}$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\,d\theta$

b) $\int x\cos(2x^2+1)\,dx = \int \cos u\,\frac{du}{4} = \frac{1}{4}\sin u + c = \frac{1}{4}\sin(2x^2+1) + C$

$u = 2x^2+1$

$du = 4x\,dx$

c) $\int \tan^3 x\,dx = \int \tan x\cdot\tan^2 x\,dx$

$= \int \tan x\,(\sec^2 x - 1)\,dx \quad \left(\tan x = \frac{\sin x}{\cos x}\right)$

$= \int \tan x\sec^2 x\,dx - \int \tan x\,dx$

$u = \tan x$, $du = \sec^2 x\,dx$: $= \int u\,du + \int\frac{dv}{v} = \frac{u^2}{2} + C + \ln|v|$

$= \frac{\tan^2 x}{2} + \ln|\cos x| + C$

$v = \cos x$

$dv = -\sin x\,dx$