MAP 2302 (Differential Equations) —Answers
QUIZ 6, Friday March 2, 2018

Name: **PID:**

1. [10] Use the method of of variation of parameters to solve the initial-value problem:
   $y'' + y = \sec x, \quad y(0) = -2, \quad y'(0) = 3$

Homogeneous D.E: $y'' + y = 0$

Auxiliary equation: $m^2 + 1 = 0 \rightarrow m_1 = i, \ m_2 = -i$

$$y_c = C_1 \cos(x) + C_2 \sin(x), \quad C_1, C_2 = \text{constants}$$

Seek $y_p = v_1(x)\cos(x) + v_2(x)\sin(x),$ with

$$v_1' \cos(x) + v_2' \sin(x) = 0 \qquad (1)$$

$$-v_1' \sin x + v_2' \cos x = \sec x \qquad (2)$$

$\sin(x) \cdot (1) + \cos(x) \cdot (2)$ yields:

$$v_1' \sin x \cos x - v_1' \sin x \cos x + v_2'(\sin^2 x + \cos^2 x) = \cos x \sec x = 1$$

$\underbrace{}_{1}$

So $v_2' = 1$; hence $v_2(x) = \int dx = x$

$(1) \rightarrow v_1' = -v_2' \dfrac{\sin x}{\cos x} = -\dfrac{\sin x}{\cos x}$

So $v_1(x) = \int -\dfrac{\sin x}{\cos x} dx = \ln|\cos x|$

Hence $y_p = \cos(x) \ln|\cos(x)| + x \sin x$

General soln: $y = C_1 \cos(x) + C_2 \sin(x) + \cos(x)\ln|\cos(x)| + x \sin x$

$y'(x) = -C_1 \sin(x) + C_2 \cos(x) - \sin(x)\ln|\cos(x)| - \sin x + \sin x$
$\qquad\qquad + x \cos x$

$y(0) = C_1 + C_2(0) + \ln(1) + 0,$ and $\ln(1) = 0, \ \sin(0) = 0$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \cos(0) = 1$

$\qquad = C_1;$ so $C_1 = -2$

$y'(0) = C_2;$ so $C_2 = 3$

Solution of IVP: $y = -2\cos x + 3\sin(x) + \cos(x)\ln|\cos(x)| + x \sin x$