MAC 2312 (Calculus II) — Answers
QUIZ 8, Friday October 21, 2016

Name: **PID:**

Remember to show all your work; you won't get any credits if only your answers are shown without the steps leading to them.

1. [3] Use the comparison test to show that the series $\sum_{k=1}^{\infty} \frac{1}{2k^2+3}$ converges.

For every $k \geq 1$, $\frac{1}{2k^2+3} < \frac{1}{2k^2} < \frac{1}{k^2}$; now $\sum_{k=1}^{\infty} \frac{1}{k^2}$ converges; p-series, $p>1$ 0.5 0.5

So $\sum_{k=1}^{\infty} \frac{1}{2k^2+3}$ converges by the comparison test 1

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2. [3] Decide whether each statement is true or false. No explanation needed.

1 a) If $\lim_{k\to\infty} u_k = 0$, then the series $\sum u_k$ converges. False, by the divergence test

1 b) If $p > 0$, then the series $\sum_{k=1}^{\infty} \frac{1}{p^k}$ is a p-series. False, series is a geometric series with ratio $r = p$

1 c) The series $\sum_{k=1}^{\infty} (-1)^k 3^{-k}$ converges. True; geometric series with ratio $r = -\frac{1}{3}$, $|r| < 1$.

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3. [4] Use the integral test to determine whether the series $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}(1+k)}$ converges or diverges. If it diverges, state whether it diverges to $+\infty$ or $-\infty$.

$$\int_1^{\infty} \frac{dx}{\sqrt{x}(x+1)} = \lim_{R\to\infty} \int_1^{R} \frac{dx}{\sqrt{x}(x+1)} \quad 0.5$$

set $u = \sqrt{x}$, then $du = \frac{dx}{2\sqrt{x}}$, and $x+1 = u^2+1$

$$= \lim_{R\to\infty} \int_1^{\sqrt{R}} \frac{2\,du}{u^2+1} \quad 1$$

$$= \lim_{R\to\infty} 2\arctan u \Big]_1^{\sqrt{R}} \quad 0.5$$

$$= 2\left(\lim_{R\to\infty} \arctan\sqrt{R} - \arctan(1)\right) \quad 0.5$$

$$= 2\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = \frac{\pi}{2} \quad 0.5$$

; the improper integral converges; 0.5
So the series converges, by the integral test 0.5