MAC 2312 (Calculus II) — Answers

QUIZ 9, Friday October 28, 2016

Name: **PID:**

Remember to show all your work; you won't get any credits if only your answers are shown without the steps leading to them.

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1. [4] a) Show that the series $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^{\frac{2}{3}}}$ converges. b) Is the convergence absolute or conditional?

a) Set $u_k = \frac{1}{k^{2/3}}$, $k=1,2,\dots$. Since $k < k+1$, we have $k^{2/3} < (k+1)^{2/3}$, so $\frac{1}{(k+1)^{2/3}} < \frac{1}{k^{2/3}}$ or $u_{k+1} < u_k$, $k=1,2,\dots$. Now $\lim_{k\to\infty} u_k = \lim_{k\to\infty} \frac{1}{k^{2/3}} = \frac{1}{+\infty} = 0$

Hence $(u_k)_k$ decreases to zero. Consequently the series converges by the alternating series test.

b) $\sum_{k=1}^{\infty} \frac{1}{k^{2/3}}$ is a p-series with $p = \frac{2}{3} < 1$; so $\sum_{k=1}^{\infty} \frac{1}{k^{2/3}}$ diverges.

Hence the convergence in a) is conditional.

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2. [2] Find an upper bound on the absolute error that results when the sum of the series $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^{\frac{2}{3}}}$ is approximated by the 7999^{th} partial sum.

Set $S = \sum_{k=1}^{\infty} \frac{(-1)^k}{k^{2/3}}$, $S_{7999} = \sum_{k=1}^{7999} \frac{(-1)^k}{k^{2/3}}$

By the alternating series Test, we know that the absolute error satisfies

$$|S_{7999} - S| \leq \frac{1}{(8000)^{2/3}} = \frac{1}{8^{2/3}(1000)^{2/3}} = \frac{1}{2^2(10)^2} = \frac{1}{400}$$

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3. [4] Let $f(x) = \cos(3x)$. Find the fourth order Taylor polynomial for f about $x = \pi/3$.

$f'(x) = -3\sin(3x)$, $f''(x) = -9\cos(3x)$, $f'''(x) = 27\sin(3x)$,
$f^{(4)}(x) = 81\cos(3x)$. $f(\pi/3) = \cos(\pi) = -1$, $f'(\pi/3) = 0$, $f''(\pi/3) = 9$, $f^{(3)}(\pi/3) = 0$, $f^{(4)}(\pi/3) = 81$

$$P_4(x) = f(\tfrac{\pi}{3}) + f'(\pi/3)(x-\tfrac{\pi}{3}) + \frac{f''(\pi/3)}{2}(x-\tfrac{\pi}{3})^2 + \frac{f^{(3)}(\pi/3)}{6}(x-\tfrac{\pi}{3})^3 + \frac{f^{(4)}(\pi/3)}{24}(x-\tfrac{\pi}{3})^4$$

$$= -1 + 0 + \frac{9}{2}(x-\tfrac{\pi}{3})^2 + 0 - \frac{81}{24}(x-\tfrac{\pi}{3})^4$$

$$= -1 + \frac{9}{2}(x-\tfrac{\pi}{3})^2 - \frac{27}{8}(x-\tfrac{\pi}{3})^4$$