MAC 2313 (Calculus III) U03
Test 1, Thursday January 31, 2008

Name: **PID:**

Remember that no documents or graphing calculators are allowed during the test. Be as precise as possible in your work; you shall show all your work to deserve the full mark assigned to any question. Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University Administration. Good luck.

1. [9] Discuss each limit.

a) $\displaystyle\lim_{(x,y,z)\to(1,2,-1)} \frac{5xy-3z}{1+2x^2+y^2-z^2}$

b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{9x^2+4y^2}$

2. [15] Let $f(x,y) = \begin{cases} \dfrac{x^{\frac{9}{5}}y}{(x^2+y^2)^{\frac{3}{4}}}, & (x,y)\neq(0,0), \\ 0, & (x,y)=(0,0). \end{cases}$

a) Is f continuous at $(0,0)$? b) Find $f_x(0,0)$, and $f_y(0,0)$. c) Is f differentiable at $(0,0)$?

3. [10+4+6] a) Show that if f is differentiable at (x_0, y_0), then f is continuous at (x_0, y_0). b) Convert the point $(1, \sqrt{3}, -2)$ from rectangular to spherical coordinates. c) Convert the equation of the surface $\rho = 2 \csc \phi$, from spherical to rectangular coordinates, and identify the surface.

4. [9+6] a) If $\tan(w) + ze^{(yx)} = w$, use implicit differentiation to find w_x, w_y, and w_z. b) If $f(x, y, z) = xy + yz + zx$, find a local linear approximation L of the function f at $P(1, -1, 1)$, and use it to approximate $f(0.99, -1.01, 1.001)$.

5. [8] If $x = uv$, $y = u^2 - v^2$, and $z = \cos(x/y)$, use the chain rule to find z_u, z_v. Express your answers in terms of u and v only.

6. [16] Consider the surface $z - 2y^2 - x^2 = 1$. a) What equation results when the surface is reflected about $x = z$? b) Describe in words the level surfaces: $x^2 + 4y^2 - z^2 = k$, for $k = -1,\ 0,\ 1$. c) Write down the equation of the sphere having the points $P(1, -1, 1)$ and $Q(1, 3, 1)$ as a diameter.

7. [20] Let $f(x, y) = y^3 - 6x^2 + 2y^2 + x^4$. Find all the critical points of f and classify each of them as a local maximum, a local minimum, or a saddle point.