MAC 2313 (Calculus III)
Test 1, Friday February 24, 2012

Name: **PID:**

Remember that no documents or calculators are allowed during the test. Be as precise as possible in your work; you shall show all your work to deserve any of the points assigned to any question. *You will not get any credit by just writing down the answer to any of the problems.* **Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University Administration. Good luck.**

1. [6] Identify the surface $x^2+y^2-2z^2=1$, and convert its equation from rectangular coordinates to spherical coordinates.

2. [12] Let $\overrightarrow{u}=-3\overrightarrow{j}+4\overrightarrow{k}$, and $\overrightarrow{v}=\overrightarrow{i}+\overrightarrow{j}-2\overrightarrow{k}$. a) Find the component of $\overrightarrow{v}$ that is parallel to $\overrightarrow{u}$, and the component of $\overrightarrow{v}$ that is orthogonal to $\overrightarrow{u}$. b) Find the direction cosines of $\overrightarrow{v}$.

3. [16] Consider the surface $4z^2+y^2-x^2=4$. a) Find an equation for the tangent plane to that surface at the point $Q(-1,1,-1)$. b) Find the parametric equations of the normal line to that surface at Q. c) Let $g(x,y,z)=xy-yz+zx+3$. Find the gradient of g at (1,-1,-1). d) Find the directional derivative of g at $(1,0,1)$ in the direction of the vector $\overrightarrow{a}=-2\overrightarrow{i}+5\overrightarrow{j}$.

4. [15] Let $f(x, y) = x^2 - 2xy + 4y^3 - 1$. Find all the critical points of f and classify them.

5. [15] a) Write down the definition of " f is differentiable at (x_0, y_0)".

b) Use the definition in a) to show that the function f given by $f(x, y) = xy - 3x$ is differentiable at the point $(-2, 1)$.

6. [10] Use the chain rule to find z_u and z_v if $z = \ln(1 + xy)$, $x = u - v$, $y = uv$. You won't get any credit if you do not use the chain rule.

7. [10] Decide whether the statement is true or false. No explanation is needed.
a) If f is differentiable at (x_0, y_0), then f is continuous at (x_0, y_0).

b) If $\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$, then $f(x,y) \to L$ as (x,y) approaches (x_0, y_0) along the line $y = y_0$ and $f(x,y) \to L$ as (x,y) approaches (x_0, y_0) along the parabola $y = y_0 + (x - x_0)^2$.

c) $\overrightarrow{u} \times \overrightarrow{v}$ is orthogonal to both $\overrightarrow{u}$ and $\overrightarrow{v}$.

d) The equation $y = x^2$ represents a parabola in 3-space.

e) In 3-space, if two lines are not parallel, then they intersect.

8. [16] Show that the two lines $L_1 : x = 1 + t, \quad y = 2 - 3t, \quad z = 1$, and $L_2 : x = 2 + t, \quad y = 1 - t, \quad z = 2 + t$
a) intersect, and find their point of intersection A.
b) Find the equation of the plane $\mathcal{P}$ that contains both L_1 and L_2.