MAC 2311 (Calculus I)
TEST 1 Review

Name: **PID:**

Remember that no documents or calculators are allowed during the exam. Be as precise as possible in your work; you shall show all your work to deserve the full mark assigned to any question. Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University Administration. Good Luck.

1. [30] Evaluate the following limits (Show all your work. You will not get any credit(s) by guessing the correct answer(s). You cannot use de l'Hopital's rule. If a limit is infinite, clearly state whether it is $+\infty$ or $-\infty$.)

a) $\lim_{x \to -1} \frac{x^2 - 3x}{x^3 - 2x + 6} =$

b) $\lim_{x \to +\infty} \frac{-5x^5 + 3x + 7}{8 - 5x^2 + 2x^4} =$

c) $\lim_{x \to -3^-} \frac{1 - x}{x + 3} =$

d) $\lim_{x \to -2} \frac{\sqrt{-2 - 3x} - 2}{x + 2} =$

e) $\lim_{x \to 3} \sqrt{\frac{3x^2 - 5x + 4}{5x - 7}} =$

f) $\lim_{x \to \frac{3}{2}^+} \frac{1}{|-2x + 3|} =$

g) $\lim_{x \to 1} \frac{x^{12} - 1}{x^3 - 1} =$

h) $\lim_{x \to +\infty} \frac{\sqrt{6x^2 - 5x + 6}}{-5x + 7} =$

i) $\lim_{x \to 1} (x^3 - 7x + 2) =$

j) $\lim_{x \to -\infty} (\sqrt{9x^2 - 5x} + 3x) =$

2. [5] If $f(x) = \begin{cases} x^3 + 3, & x \geq -2 \\ 3x + 1, & x < -2. \end{cases}$
Is f continuous at $x = -2$? You must carefully explain your answer to get any credits.

3. [5] Use the rigorous definition of limit to prove that $\lim_{x \to 5} (-3x + 10) = -5$.

4. [5] Express $f(x) = |-5x + 9| - |3x + 8|$ in a piecewise defined form without using the absolute value symbol.

5. [5] a) State the intermediate value theorem. b) Use it to show that the equation $2x^{712} - 7x^7 + 1 = 0$ has a solution in the open interval $(0, 1)$.

6. [30] Evaluate the following limits (Show all your work. You cannot use de l'Hopital's rule for any of the limits, otherwise you'll get a zero. You will not get any credit(s) by guessing the correct answer(s). If a limit is infinite, clearly state whether it is $+\infty$ or $-\infty$.)

a) $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{x^2 + x} \right) =$

b) $\lim_{x \to -\infty} \frac{5x^5 - 7x^3 + 10x + 10^{12}}{2x - x^4 + 5} =$

c) $\lim_{x \to 2\pi} \frac{\sin x}{x} =$

d) $\lim_{x \to 1} \frac{\sqrt{x} - 1}{x^2 - 1} =$

e) $\lim_{x \to -5^-} \frac{x + 3}{x + 5} =$

f) $\lim_{x \to 0} \frac{\sin^2(4x)}{x^2} =$

g) $\lim_{x \to 2} \frac{\cos(\frac{\pi}{x})}{x - 2} =$

h) $\lim_{x \to \frac{\pi}{4}} \frac{\cos x - \sin x}{x - \frac{\pi}{4}} =$

i) $\lim_{x \to 1} \frac{\sin(\pi x)}{x - 1} =$

j) $\lim_{x \to +\infty} x^2(1 - \cos(1/x)) =$

7. [5] a) Write down the rigorous definition of $\lim_{x \to -3} f(x) = L$. b) Use that definition to show that $\lim_{x \to -3} (-4x + 1) = 13$.

8. [5] Decide whether the statement is true or false. No explanation needed.

a) If f is continuous at x_0, then $\lim_{x\to x_0} f(x) = f(x_0)$.

b) If $f(-3) = 5$, then $\lim_{x\to -3} f(x) = 5$.

c) If $\lim_{x\to x_0^+} f(x) = 26$ and $\lim_{x\to x_0^-} f(x) = 26$, then f is continuous at x_0.

d) If $|f|$ is continuous at -1, then f is continuous at -1.

e) $\lim_{x\to +\infty} (x - x^2) = +\infty - (+\infty) = 0$.

9. [5] Sketch a possible graph for a function f satisfying the following properties:
i) $f(-3) = f(0) = f(2) = 0$
(ii) $\lim_{x\to -2^+} f(x) = -\infty$ and $\lim_{x\to -2^-} f(x) = +\infty$
(iii) $\lim_{x\to 1} f(x) = +\infty$.

10. [5] Find all values of x at which the given function is continuous: a) $f(x) = \sin^{-1}(\ln(2x))$, b) $g(x) = \log_{3x+4}(-5x+12)$, c) $h(x) = \ln(-2x+7) - \ln(4x+5)$, d) $k(x) = \log_x \left(\frac{3-2x}{5x+8}\right)$.

1. [32] Find the derivative of each of the following functions (Show all your work, and simplify your answers as much as possible; you will not get any credit(s) by guessing the correct answer(s).)

a) $f(x) = \frac{2x}{x^2-3x+1}$

b) $p(x) = (2x+7)^9(3x-5)^6$

c) $h(x) = \tan(x^4)$

d) $k(x) = \cos(2x)\tan(x)$

e) Use the implicit differentiation technique to find $\frac{dy}{dx}$ if $xy + x^2\cos(y) = 1$.

f) $m(x) = \frac{1+x^2}{\cot x + \csc x}$

g) Find all values of x at which the line that is tangent to $y = 3x - \tan x$ is parallel to the line $y - x = 2$.

2. [6] Use the definition of the derivative to evaluate the limits

a) $\lim_{x \to 2} \frac{\sec(\pi x/8) - \sqrt{2}}{x-2} =$

b) $\lim_{x \to 1} \frac{x^{11} - 1}{x - 1} =$

3. [5] Let $y = 2x^2 - 3$. a) Find the average rate of change of y with respect to x on the interval $[-1, 2]$. b) Find the instantaneous rate of change of y with respect to x at $x_0 = -1$.

4. [5] a) Write down the two definitions for $f'(x_0)$. b) Use any of those definitions to find $f'(1)$ if $f(x) = \sqrt{x}$. c) Use part b) to find the equation of the tangent line to the curve $y = \sqrt{x}$ at $x = 1$.

5. [5] If $f(x) = \begin{cases} 3x^2 - 5, & x > -1 \\ 5x^3 + 3, & x \leq -1. \end{cases}$

a) Show that f is continuous at $x = -1$. b) Is f differentiable at $x = -1$? You must carefully explain your answer to get any credits.

6. [34] Find the derivative of each of the following functions (Show all your work, and simplify your answers as much as possible; you will not get any credit by guessing the correct answer(s).)

a) $f(x) = -4x^3 - \frac{8}{\sqrt[4]{x}} + \frac{7}{x^5} - e^{-6}$

b) $g(x) = \frac{4x-5}{x^2+x+1}$

c) $h(x) = x^3 \sin(x^2)$

d) $k(x) = \sec^2(\sin x) - \tan^2(\sin x)$

e) $l(x) = \sin(3x) - 3\tan(\cos x)$

f) $m(x) = \cos(\cos x)$

g) Suppose that a function f is differentiable at $x = 2$, and $\lim_{x \to 2} \dfrac{x^3 f(x) - 24}{x - 2} = 28$. Find $f(2)$ and $f'(2)$.

h) Find all values of x at which the tangent line to the curve $y = 2x^3 - x^2$ is perpendicular to the line $x + 4y = 10$.

7. [10] Decide whether the statement is true or false. No explanation needed.

a) If $f(x) = \frac{\sin x}{g(x)}$, then $f'(x) = \frac{\cos x}{g'(x)}$.

b) $\frac{d}{dx}(\sin(f(x))) = \cos(f'(x))$.

c) If $f(x) = h(\cos x)$, then $f'(x) = h'(-\sin x)$.

d) If f is differentiable at -7, then f is continuous at -7.

e) If f is continuous at 2, then f is differentiable at 2.

f) If $g(x) = \tan(2x)$, then $g'(x) = \sec^2(2x)$.

g) If $k(x) = \cos^2(x^3) + \sin^2(x^3)$, then $k'(x) = 0$.

h) If $p(x) = f(x) \tan x$, then $p'(x) = f'(x) \sec^2 x$.

i) If $\lim_{x \to -1} \frac{f(x) - f(-1)}{x+1} = -2$, then $\lim_{h \to 0} \frac{f(-1+h) - f(-1)}{h} = -2$.

j) If $m(x) = e^6$, then $m'(x) = 6e^5$.