MAC 2311 (Calculus I) – Answers
TEST 1, Wednesday September 16, 2015

Name: **PID:**

Remember that no documents or calculators are allowed during the exam. Be as precise as possible in your work; you shall show all your work to deserve the full mark assigned to any question. Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University Administration. You may show your work on the back of page. Good Luck.

1. [24] Evaluate the following limits (Show all your work. You will not get any credit(s) by guessing the correct answer(s). You cannot use de l'Hopital's rule for any of the limits, otherwise you'll get a zero. If a limit is infinite, clearly state whether it is $+\infty$ or $-\infty$.)

a) $\lim_{x\to -3}\frac{5-4x}{x^2+3x-5} = \frac{5-4(-3)}{9-9-5} = \frac{17}{-5} = -\frac{17}{5}$

b) $\lim_{x\to -\infty}\frac{\sqrt{5x^2-7x+3}}{7+3x} = \lim_{x\to-\infty}\frac{\sqrt{5x^2}}{3x} = \lim_{x\to-\infty}\frac{\sqrt{5}(-\cancel{x})}{3\cancel{x}} = -\frac{\sqrt{5}}{3}$

c) $\lim_{x\to 2^+}\frac{1-x}{x^2-4} = \lim_{x\to 2^+}\frac{1-x}{(x-2)(x+2)} = \lim_{x\to 2^+}\frac{(1-x)}{x+2}\cdot\lim_{x\to 2^+}\frac{1}{x-2} = -\frac{1}{4}(+\infty) = -\infty$

d) $\lim_{x\to -1}\frac{2-\sqrt{1-3x}}{x+1} = \lim_{x\to-1}\frac{(2-\sqrt{1-3x})(2+\sqrt{1-3x})}{(x+1)(2+\sqrt{1-3x})} = \lim_{x\to-1}\frac{4-(1-3x)}{(x+1)(2+\sqrt{1-3x})} = \lim_{x\to-1}\frac{3\cancel{(1+x)}}{\cancel{(x+1)}(2+\sqrt{1-3x})} = \frac{3}{4}$

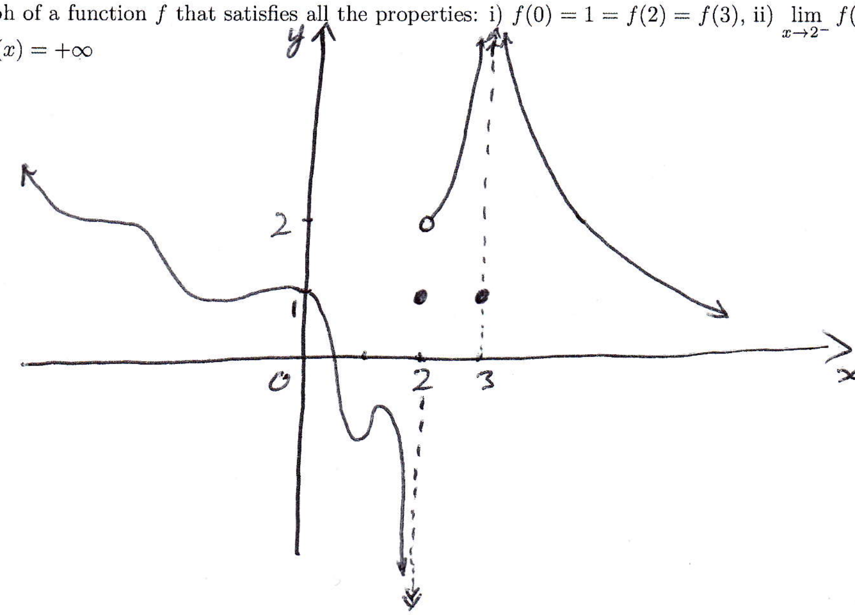
e) $\lim_{x\to 0}\frac{\sin(3x)}{\sin(12x)} = \lim_{x\to 0}\frac{\frac{\sin(3x)}{3x}}{\frac{\sin(12x)}{12x}}\cdot\frac{3\cancel{x}}{12\cancel{x}} = \frac{1}{4}\cdot\frac{1}{1} = \frac{1}{4}$

f) $\lim_{x\to\frac{\pi}{6}}\frac{\sin x}{x} = \frac{\sin\frac{\pi}{6}}{\frac{\pi}{6}} = \frac{6}{\pi}(1/2) = \frac{3}{\pi}$

g) $\lim_{x\to 1}\frac{x^3-1}{x^2+2x-3} = \lim_{x\to 1}\frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}(x+3)} = \frac{3}{4}$

h) $\lim_{x\to+\infty}(\sqrt{4x^2-7x+11}-2x) = \lim_{x\to+\infty}\frac{(\sqrt{4x^2-7x+11}-2x)(\sqrt{4x^2-7x+11}+2x)}{\sqrt{4x^2-7x+11}+2x} = \lim_{x\to+\infty}\frac{\cancel{4x^2}-7x+11-\cancel{4x^2}}{\sqrt{4x^2}+2x} = \lim_{x\to+\infty}\frac{-7x}{2x+2x} = \lim_{x\to+\infty}\frac{-7x}{4x} = -\frac{7}{4}$

2. [6] Sketch the graph of a function f that satisfies all the properties: i) $f(0)=1=f(2)=f(3)$, ii) $\lim_{x\to 2^-}f(x)=-\infty$ $\lim_{x\to 2^+}f(x)=2$, $\lim_{x\to 3}f(x)=+\infty$



3. [10, **Bonus**] Let $f(x) = \begin{cases} 2x^2 - 5 \text{ if } x < -2 \\ -2x - 1 \text{ if } x \geq -2. \end{cases}$

Find a) $f(-2)$, b) $\lim_{x \to -2^-} f(x)$, c) $\lim_{x \to -2^+} f(x)$, d) $\lim_{x \to -2} f(x)$. Is f continuous at $x = -2$? Show all your work.

a) $f(-2) = -2(-2) - 1 = 3$

b) $\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} (2x^2 - 5) = 2(4) - 5 = 3$

c) $\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} (-2x - 1) = -2(-2) - 1 = 3$

d) $\lim_{x \to -2} f(x) = 3$ since $\lim_{x \to -2^-} f(x) = 3 = \lim_{x \to -2^+} f(x)$

Since $f(-2) = 3 = \lim_{x \to -2} f(x)$, f is continuous at $x = -2$.

4. [10] a) Write down the rigorous definition of $\lim_{x \to -3} f(x) = 11$. b) Use the rigorous definition of limit to prove that $\lim_{x \to -3} (2 - 3x) = 11$.

a) $\forall \varepsilon > 0, \ \exists \delta > 0 : \forall x, \ 0 < |x + 3| < \delta \Rightarrow |f(x) - 11| < \varepsilon.$

b) Let $\varepsilon > 0$. Find $\delta > 0 : \forall x, \ 0 < |x + 3| < \delta \Rightarrow |2 - 3x - 11| < \varepsilon$.

For $|2 - 3x - 11| < \varepsilon$, it suffices that $|-3x - 9| < \varepsilon$

For $|-3(x+3)| < \varepsilon$, it suffices that $|-3||x+3| < \varepsilon$

or $3|x+3| < \varepsilon$

or $|x+3| < \varepsilon/3$

We may choose $\delta = \varepsilon/3$.

5. [10] a) State the intermediate value theorem (IVT0). b) Use it to show that the equation $x^{916} - 6x^6 + 2 = 0$ has a solution in the open interval $(-1, 0)$.

a) If a function f is continuous on $[a, b]$, and $f(a)f(b) < 0$, then there exists x_0 in (a, b) such that $f(x_0) = 0$.

b) Set $f(x) = x^{916} - 6x^6 + 2$. Then f is continuous on $[-1, 0]$ since f is a polynomial. On the other hand

$f(-1) \cdot f(0) = (-3)(2) < 0$.

Therefore, there exists x_0 in $(-1, 0)$ with $f(x_0) = 0$, by IVTϕ.