MAC 2311 (Calculus I)
TEST 1 Review

Name: **PID:**

Remember that no documents or calculators are allowed during the exam. Be as precise as possible in your work; you shall show all your work to deserve the full mark assigned to any question. Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University Administration. Good Luck.

1. [30] Evaluate the following limits (Show all your work. You will not get any credit(s) by guessing the correct answer(s). If a limit is infinite, clearly state whether it is $+\infty$ or $-\infty$.)

a) $\displaystyle\lim_{x\to -1} \frac{x^2-3x}{x^3-2x+6} =$

b) $\displaystyle\lim_{x\to +\infty} \frac{-5x^5+3x+7}{8-5x^2+2x^4} =$

c) $\displaystyle\lim_{x\to -3^-} \frac{1-x}{x+3} =$

d) $\displaystyle\lim_{x\to -2} \frac{\sqrt{-2-3x}-2}{x+2} =$

e) $\displaystyle\lim_{x\to 3} \sqrt{\frac{3x^2-5x+4}{5x-7}} =$

f) $\displaystyle\lim_{x\to \frac{3}{2}^+} \frac{1}{|-2x+3|} =$

g) $\displaystyle\lim_{x\to 1} \frac{x^{12}-1}{x^8-1} =$

h) $\displaystyle\lim_{x\to +\infty} \frac{\sqrt{6x^2-5x+6}}{-5x+7} =$

i) $\displaystyle\lim_{x\to 1} (x^3-7x+2) =$

j) $\displaystyle\lim_{x\to -\infty} (\sqrt{9x^2-5x}+3x) =$

2. [5] If $f(x) = \begin{cases} x^3+3, & x \geq -2 \\ 3x+1, & x < -2. \end{cases}$

Is f continuous at $x = -2$? You must carefully explain your answer to get any credits.

3. [5] Use the rigorous definition of limit to prove that $\lim_{x \to +\infty} \frac{2x+1}{3x-4} = \frac{2}{3}$.

4. [5] Express $f(x) = |-5x+9| - |3x+8|$ in a piecewise defined form without using the absolute value symbol.

5. [5] a) State the intermediate value theorem. b) Use it to show that the equation $2x^{712} - 7x^7 + 1 = 0$ has a solution in the open interval $(0, 1)$.

6. [30] Evaluate the following limits (Show all your work. You cannot use de l'Hopital's rule for any of the limits, otherwise you'll get a zero. You will not get any credit(s) by guessing the correct answer(s). If a limit is infinite, clearly state whether it is $+\infty$ or $-\infty$.)

a) $\lim\limits_{x\to 0}\left(\dfrac{1}{x}-\dfrac{1}{x^2+x}\right)=$

b) $\lim\limits_{x\to -\infty}\dfrac{5x^5-7x^3+10x+10^{12}}{2x-x^4+5}=$

c) $\lim\limits_{x\to 2\pi}\dfrac{\sin x}{x}=$

d) $\lim\limits_{x\to 1}\dfrac{\sqrt{x}-1}{x^2-1}=$

e) $\lim\limits_{x\to -5^-}\dfrac{x+3}{x+5}=$

f) $\lim\limits_{x\to 1}\dfrac{3x^4-4x+1}{x^3-1}=$

g) $\lim\limits_{x\to -2}\dfrac{4x^3+19x^2+28x+12}{2x^3+3x^2-12x-20}=$

h) $\lim\limits_{x\to -\infty}\sqrt[3]{\dfrac{8x^5-4x^2+7}{5-6x^3-27x^5}}=$

i) $\lim\limits_{x\to 0}\sqrt[5]{\dfrac{5x^2-7x+32}{8x^3-9x-1}}=$

j) $\lim\limits_{x\to 4}\dfrac{\sqrt{5x-4}-4}{x^3-x^2-13x+4}=$

7. [5] a) Write down the rigorous definition of $\lim\limits_{x\to -3}f(x)=L$. b) Use that definition to show that $\lim\limits_{x\to -3}(-4x+1)=13$.

8. [5] Decide whether the statement is true or false. No explanation needed.

a) If f is continuous at x_0, then $\lim_{x \to x_0} f(x) = f(x_0)$.

b) If $f(-3) = 5$, then $\lim_{x \to -3} f(x) = 5$.

c) If $\lim_{x \to x_0^+} f(x) = 26$ and $\lim_{x \to x_0^-} f(x) = 26$, then f is continuous at x_0.

d) If $|f|$ is continuous at -1, then f is continuous at -1.

e) $\lim_{x \to +\infty} (x - x^2) = +\infty - (+\infty) = 0$.

9. [5] Sketch a possible graph for a function f satisfying the following properties:
   i) $f(-3) = f(0) = f(2) = 0$
   (ii) $\lim_{x \to -2^+} f(x) = -\infty$ and $\lim_{x \to -2^-} f(x) = +\infty$
   (iii) $\lim_{x \to 1} f(x) = +\infty$.

10. [5] Find all values of x at which the given function is continuous: a) $f(x) = \sin^{-1}(\ln(2x))$, b) $g(x) = \log_{3x+4}(-5x+12)$, c) $h(x) = \ln(-2x+7) - \ln(4x+5)$, d) $k(x) = \log_x \left(\frac{3-2x}{5x+8}\right)$.