MAC 2313 (Calculus III)

Test 1 Review. Test 1 will cover sections 1 to 6 in chapter 11.

1. Describe the given surface; if it is a sphere, state its radius and center. If it is a point, state its coordinates. a) $x^2 + y^2 + z^2 + 6x - 2y - 6 = 0$. b) $x^2 + y^2 + z^2 - 2mx - 6y - 8z + 50 = 0$, where m is a parameter, (discuss according to the values of m).

2. a) Find an equation for the sphere passing through the origin and centered at the point $C(1, -2, 5)$. b) Decide whether the points $A(2, 3, 1)$, $B(-1, 1, -2)$ and $C(1, -1, 1)$ are the vertices of an equilateral triangle.

3. Let $\vec{r}$=2$\vec{i}$-3$\vec{j}$+4$\vec{k}$, $\vec{z}$=3$\vec{j}$-5$\vec{k}$, and $\vec{v}$=-2$\vec{i}$+$\vec{j}$-4$\vec{k}$. a) Find the area of the parallelogram having $\vec{r}$ and $\vec{z}$ as adjacent sides. b) Find the volume of the parallelepiped having $\vec{r}$, $\vec{z}$ and $\vec{v}$ as adjacent edges. c) Find the acute angle θ between $\vec{v}$ and the plane containing the face determined by $\vec{r}$ and $\vec{z}$.

4. Consider the lines: $L_1 : x = 4 - 2t, \quad y = 2 + 3t, \quad z = 1 + t$ and $L_2 : x = 2 + 4t, \quad y = 3 - 6t, \quad z = -2t$. a) Show that L_1 and L_2 are parallel lines. b) Find the distance between L_1 and L_2.

5. a) Let $A(x_0, y_0, z_0)$ be a given point in 3-space. Let $\mathcal{P}$ be the plane with equation $ax + by + cz + d = 0$. Write down the distance D between A and the plane $\mathcal{P}$. $D =$
b) Use a) to find the distance between the two skew lines: $L_1 : x = -2 + t, \quad y = 3 + 2t, \quad z = 1 + 8t$ and $L_2 : x = 1 - 2t, \quad y = -2 + 3t, \quad z = -1 + 5t$.

6. Let $\vec{w}$=$\vec{i}$-2$\vec{j}$+3$\vec{k}$ and $\vec{v}$=2$\vec{i}$-$\vec{j}$-5$\vec{k}$. a) Find the vector component of $\vec{v}$ that is parallel to $\vec{w}$ and the vector component of $\vec{v}$ that is orthogonal to $\vec{w}$. b) If θ denotes the angle between $\vec{v}$ and $\vec{w}$, find $\cos(\theta)$ and $\sin(\theta)$. Is θ acute or obtuse? c) Find the direction angles of $\vec{w}$.

7. a) Set $\vec{u}$=$\vec{i}$-3$\vec{k}$, $\vec{v}$=-$\vec{j}$+$\vec{k}$ and $\vec{w}$=2$\vec{i}$-$\vec{j}$. Let $\vec{z}$=$\vec{i}$-$\vec{j}$+2$\vec{k}$. Find scalars a, b, and c such that $\vec{z} = a\vec{u} + b\vec{v} + c\vec{w}$. b) If we now set: $\vec{u}$=$\vec{i}$+$\vec{j}$-2$\vec{k}$, $\vec{v}$=$\vec{i}$+$\vec{j}$+$\vec{k}$ and $\vec{w}$=$\vec{i}$-$\vec{j}$, find scalars α, β and γ such that $\vec{z} = \alpha\vec{u} + \beta\vec{v} + \gamma\vec{w}$.

8. a) Find parametric equations for the line through the points $A(-1, 2, 3)$ and $B(2, -3, 4)$. b) Find the vector $\vec{w}$ of norm 4 that is oppositely directed to $\vec{z}$=2$\vec{i}$-$\vec{j}$+3$\vec{k}$. c) Find parametric equations for the line through the point $A(5, 0, -2)$ that is parallel to the planes $x - 4y + 2z = 2$ and $2x + 3y - z + 1 = 0$. d) Find an equation for the plane through the points $A(-2, 1, 4)$, $B(1, 0, 3)$ that is perpendicular to the plane $4x - y + 3z = -1$. c) Let L be the line defined by the parametric equations $x = 1 - 2t,\ y = 2 + 3t,\ z = 3 + t$. Let $\mathcal{P}$ be the plane defined by $2x + y - z = 4$. c1) Show that L and $\mathcal{P}$ are not perpendicular to each other. c2) Find an equation for the plane $\mathcal{Q}$ that both contains L and is perpendicular to $\mathcal{P}$.

9. a) Show that the two lines $L_1 : x = 1 - t, \quad y = 2 + t, \quad z = 1 + 5t$, and $L_2 : x = 2 + t, \quad y = 2 + 3t, \quad z = -1 + 7t$ intersect, and find their point of intersection A. b) Find the acute angle θ between L_1 and L_2 at A. c) Find an equation for the plane that contains both L_1 and L_2. d) Find an equation for the plane that contains both L_1 and the point $B(1, -2, -1)$.

10. a) If a bug walks on the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 3 = 0$, how close and how far can it get to the origin? b) The distance between the point $P(x, y, z)$ and the point $A(1, -2, 0)$ is twice the distance between P and the point $B(0, 1, 1)$. Show that the set of all such points is a sphere, and find its center and radius.

11. a) Find an equation for the plane $\mathcal{P}$ that contains the line $L : x = 3t, \quad y = 1 + t, \quad z = 2t$, and is parallel to the intersection of the planes $y + z = -1$ and $2x - y + z = 6$. b) Show that the lines $L_1 : x = -2 + t, \quad y = 3 + 2t, \quad z = 4 - t$ and $L_2 : x = 3 - t, \quad y = 4 - 2t, \quad z = t$ are parallel, and find an equation for the plane they determine. c) Find the distance between L_1 and L_2.

12. Let $\vec{u}$ and $\vec{v}$ be adjacent sides of a parallelogram. Use vectors to show that the parallelogram is a rectangle if the diagonals are equal in length.

13. Prove that for any vectors $\vec{u}$ and $\vec{v}$, one has $\vec{u} \cdot \vec{v} = \frac{1}{4}||\vec{u} + \vec{v}||^2 - \frac{1}{4}||\vec{u} - \vec{v}||^2$.

14. Review all the true/false problems from chapter 11 in the text.