MAC 2313 (Calculus III)-U03
Test 2, Thursday February 28, 2008

Name: **PID:**

Remember that no documents or graphing calculators are allowed during the test. Be as precise as possible in your work; you shall show all your work to deserve the full mark assigned to any question. Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University Administration. Good luck.

1. [26] Evaluate each integral.

a) $\int_{-1}^{1}\int_{x}^{x^2}\int_{0}^{\ln z} xe^y \, dydzdx =$

b) $\int_{0}^{1}\int_{1}^{2} (x-2z)\, dxdz =$

c) $\int_{\frac{1}{2}}^{1}\int_{x}^{\frac{1}{x}} x \, dydx =$

d) $\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{0}^{\sqrt{1-x^2-y^2}} (x^2+y^2+z^2)^{\frac{-3}{4}} \, dzdxdy =$

2. [15] Use polar coordinates to evaluate $\int_0^1 \int_{x^2}^{x} \sqrt{x^2 + y^2}\, dydx$. (Hint. First sketch the region of integration.)

3. [18] Let $x = u - v + w$, $y = v - w$, $z = w - u$. Find the jacobians $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ and $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

4. [15] Let $\overrightarrow{u}=2\overrightarrow{i}-3\overrightarrow{j}+\overrightarrow{k}$, and $\overrightarrow{v}=\overrightarrow{i}-\overrightarrow{j}+2\overrightarrow{k}$. a) Find the angle θ between $\overrightarrow{u}$ and $\overrightarrow{v}$ to the nearest degree. b) Find the component of $\overrightarrow{u}$ that is parallel to $\overrightarrow{v}$, and the component of $\overrightarrow{v}$ that is orthogonal to $\overrightarrow{u}$. c) Find the direction cosines of $\overrightarrow{v}$, and approximate the direction angles to the nearest degree.

5. [10] Use an appropriate order of integration to evaluate $\int\int_{\mathcal{R}} y\sin(xy)\,dA$, where $\mathcal{R}=\{(x,y); 0\le x\le 1,\ \ 0\le y\le \pi/4$

6. [16] Let $f(x, y) = x + y + z$. Use the Lagrange multipliers to find the maximum and minimum values of f subject to the constraint: $x^2 + y^2 - z^2 = 1$.