**MAC 2313 (Calculus III)**
**Test 2, Thursday October 26, 2006**

**Name:** **PID:**

**Remember that no documents or calculators are allowed during the test. Be as precise as possible in your work; you shall show all your work to deserve the full mark assigned to any question. Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University Administration. Good luck.**

1. [12] Determine whether each of the following limit exits.

a) $\lim_{(x,y)\to(0,0)} \dfrac{xy}{x^2+2y^2}$

b) $\lim_{(x,y,z)\to(0,0,0)} \dfrac{xyz}{x^2+y^2+z^2}$

2. [12] Let $f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2}, & (x,y) \neq (0,0), \\ 0, & (x,y) = (0,0). \end{cases}$

Find $f_x(0,0)$, and $f_y(0,0)$. Is $f$ differentiable at $(0,0)$?

3. [10] Consider the surface $xz + 2yz^2 - zy^2 = 1$. a) Find an equation for the tangent plane to the surface at the point $P(1, 2, 1)$. b) Find the parametric equations of the line normal to the surface at $P$.

4. [18] a) Find the velocity, the speed, and the acceleration, all of them at time $t = \pi/2$, of a particle moving along the curve $r(t) = e^t \vec{i} + e^t \sin(t) \vec{j} + e^t \cos(t) \vec{k}$. b) What is the curvature of that curve at each time $t$?

5. [9+6] a) If $w^3 + x\cos(yz) = x$, use implicit differentiation to find $w_x$, $w_y$, and $w_z$. b) If $f(x, y, z) = y\cos(xz)$, find a local linear approximation $L$ of the function $f$ at $P(0, 1, 0)$, and use it to approximate $f(0.1, 1.1, 0.1)$.

6. [15] a) If $x = uv$, $y = u^2 - v^2$, $z = u^2 + v^2$, and $w = \ln(1 + x + y + z)$, find $w_u$, $w_v$. Express your answers in terms of $u$ and $v$. b) If $f(x, y, z) = xyz$, find the directional derivative of $f$ at $P(1, 1, 1)$ in the direction from $P$ to $Q(1, 2, -1)$.

7. [20] Let $f(x, y) = -x^3 + 6x + y^4 - 2y^2$. Find all the critical points of $f$ and classify each of them as a local maximum, a local minimum, or a saddle point.