MAC 2313 (Calculus III)
Test 2, Wednesday October 19, 2016

Name: **PID:**

Remember that no documents or calculators are allowed during the test. Be as precise as possible in your work; you shall show all your work to deserve any of the points assigned to any question. *You will not get any credit if you do not show the steps to your answers*. **Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University Administration. 3 pages. Total=85 points. Always do your best.**

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1 . [15] a) Describe and sketch the largest region where the function h defined by $h(x, y) = \ln(y - x^2 - 1)$ is continuous.

b) Describe in words, the domain of the function f given by $f(x, y, z) = \sqrt{x^2 + 4y^2 - z^2}$

c) Find an equation for the level surface of the function g defined by $g(x, y, z) = \int_x^{yz} \frac{t}{t^2+1}\, dt$ that passes through the point $P(1, \sqrt{3}, 1)$.

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2. [15] a) Write down the definition of "f is differentiable at (x_0, y_0)". b) Use the definition in a) to show that the function f given by $f(x, y) = 2xy - y^2$ is differentiable at $(1, -1)$.

3. [10] Decide whether the statement is true or false. No explanation is needed.

a) If $z(t) = f(x(t), y(t))$, then $\frac{dz}{dt}(t) = f_x(\frac{dx}{dt}, y) + f_y(x, \frac{dy}{dt})$.

b) If $f_x(1,2)$ and $f_y(1,2)$ both exist, then f is continuous at $(1,2)$.

c) If f is differentiable at $(7,2,-9)$, then f is continuous at $(7,2,-9)$.

d) If $\lim_{(x,y)\to(-1,1)} f(x,y) = 3$, then $f(x,y) \to 3$ as (x,y) approaches $(-1,1)$ along the line $y = 1$ and $f(x,y) \to 3$ as (x,y) approaches $(-1,1)$ along the parabola $y = 1 + (x+1)^2$.

e) If $f = f(x,y,z)$ is differentiable at the point $B(-1,4,-5)$, then the directional derivative of f at B in the direction of the vector $\vec{r} = \frac{1}{2}(\vec{i} - \sqrt{2}\,\vec{j} + \vec{k})$ is given by $D_{\vec{r}} f(B) = \nabla f(B) \cdot \vec{r}$.

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4. [7] Evaluate each limit. If a limit does not exist, explain why.

a) $\displaystyle\lim_{(x,y,z)\to(-1,1,2)} \frac{xyz}{x^2+y^2+z^2}$

b) $\displaystyle\lim_{(u,v)\to(1,-1)} \frac{u^3+v^3}{u^2-v^2}$

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5. [20] a) Find an equation for the tangent plane and parametric equations for the normal line to the surface $3\sqrt{x^2+y^2} - 2\sqrt{y^2+z^2} = 1$ at the point (1,0,1).

b) Set $h(x,y,z) = xy\cos(yz)$ Find the local linear approximation of h about the point $P(1, 1/2, \pi)$, then, use it to approximate $h(1.01, 0.498, (0.99)\pi)$.

c) Find a unit vector in the direction in which the function h in b) increases most rapidly at the point P, and find the rate of change of h at P in that direction.

6. [12] Let $f(x,y) = x^2 + x^2y - y^2 - 4y + 1$. Find all the critical points of f and classify each of them as a local maximum, a local minimum, or a saddle point.

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7. [6] Use implicit partial differentiation to find the partial derivatives $\frac{\partial y}{\partial x}$ and $\frac{\partial y}{\partial z}$ if $yz + x\sin(xy) = 1$.