MAP 2302 (Differential Equations)
TEST 2, Thursday March 25, 2010
Name:

## PID:

Remember that no documents or calculators are allowed during the test. You shall show all your work to deserve the full mark assigned to any question. Good luck.

1. [20] a) Given that $y=\sin (2 x)$ solves the differential equation: $y^{\text {iv }}-2 y^{\prime \prime \prime}+6 y^{\prime \prime}-8 y^{\prime}+8=0$, find the general solution.
2. [20] Use the method of undetermined coefficients to solve the differential equation: $y^{\prime \prime}+6 y^{\prime}+13 y=e^{x} \sin x$
3. $[10+20]$ a) Solve the Cauchy-Euler equation: $x^{2} y^{\prime \prime}-x y^{\prime}+4 y=0$. Write the general solution in terms of the variable $x>0 . \mathrm{b}$ ) Use the method of variation of parameters to find the general solution of the differential equation: $y^{\prime \prime}+4 y^{\prime}+4 y=x^{-4} e^{-2 x}$.
4. [15] Find power series solutions in powers of $x$ of the differential equation: $y^{\prime \prime}+(x+1) y^{\prime}+y=0$. Write down the general solutions including powers of $x$ up to $x^{3}$.
5. [15] Use the method of Frobenius to find two linearly independent series solutions of the form $x^{r} \sum_{n=0}^{\infty} a_{n} x^{n}$ to the differential equation equation $2 x^{2} y^{\prime \prime}+3 x y^{\prime}+(4 x-6) y=0, \quad 0<x<R$. (First find and solve the indicial equation, then for each indicial root, find a recurrence relation between $a_{n}$, and $a_{n-1}$. This will be enough.)
