MAC 2313 (Calculus III)

Test 2 Review: Test 2 is due Friday 11/03/17, and will cover sections 13.4 to 13.9, and 14.1 to 14.3.

1. a) If $x = v\ln u$, $y = u\ln v$, use implicit partial differentiation to find u_x, v_x, u_y and v_y. If we set $z = \tan(2u-3v)$, use the chain rule to find z_x and z_y. b) answer the same questions as in c) if $x = u^2 - v^2$, $y = u^2 - v$, and $z = u^2 + v^2$.

2. Find an equation for the tangent plane and parametric equations for the normal line to the given surface at the given point. a) $2x^2 - 2y^2 + 4z^2 = 4$, $P(-1,1,1)$. b) $\dfrac{x+2y}{2y+z} = -1$, $P(1,1/4,-2)$.

3. Find parametric equations for the tangent line to the curve of intersection of the given surfaces at the given point. a) $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$, $P(\sqrt{2}, -\sqrt{2}, 4)$. b) $x^2 + y^2 = 2$ and $x + 2y + 3z = 6$, $P(1,1,1)$.

4. Let $f(x,y) = \begin{cases} \dfrac{x^{\frac{12}{5}}y}{x^2+y^2} - 2x + 3y, & (x,y) \neq (0,0), \\ 0, & (x,y) = (0,0). \end{cases}$
a) Find $f_x(0,0)$, and $f_y(0,0)$. b) Is f differentiable at $(0,0)$?

5. a) Write down the definition of " f is differentiable at (x_0,y_0)". b) Use the definition in a) to show that the function f given by $f(x,y) = 2x - 3xy$ is differentiable at the point $(1,-2)$.

6. Let $g(x,y,z) = xye^{x+yz}$. a) Find the gradient of g at $P(1,1,-1)$. b) Find a unit vector in the direction in which g decreases most rapidly at the point $Q(-1,1,1)$, and find the rate of change of g at Q in that direction. c) Find the directional derivative of g in the direction of the vector $\vec{a}=2\vec{i}-3\vec{j}+4\vec{k}$ at the point $A(1,-1,1)$.

7. Find all the critical points of f and classify them as points of local minimum, local maximum, or saddle points. a) $f(x,y) = xy + 2x - \ln(x^2y)$, b) $f(x,y) = x^3 - y^3 - 2xy + 6$, c) $f(x,y) = 4xy + x^4 + y^4$, d) $f(x,y) = 2y^2x - x^2y + 4xy$.

8. If z is defined implicitly by the relation $f(x,y,z) = 0$ as a differentiable function of x and y, where f is a differentiable function with $f_z(x,y,z) \neq 0$ for all allowable points (x,y,z), use implicit differentiation and the chain rule to find the partial derivatives z_x and z_y.

9. a) Find the point on the paraboloid $z = x^2 + y^2 + 10$ that is closest to the plane $x + 2y - z = 0$. b) Find three positive numbers whose sum is 48 and their product is as large as possible. c) Problems 44 and 48 in 13.8, p. 987.

10. Given that the directional derivative of a function f at the point $P(3,-2,1)$ in the direction of the vector $\vec{a} = 2\vec{i} - \vec{j} - 2\vec{k}$ is -5 and that $||\nabla f(P)|| = 5$, find $\nabla f(P)$. (Hint. You may use the angle between $\vec{a}$ and $\nabla f(P)$.)

11. Evaluate each integral.

a) $\int\int_R e^s \ln t \, dA$; R = region in the first quadrant of the st-plane that lies above the curve $s = \ln t$ from $t = 1$ to $t = 2$. b) $\int_0^\pi \int_x^\pi \frac{\sin y}{y}\, dydx$. c) $\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\, dydx$. d) $\int_0^{\ln 2} \int_{e^y}^2 e^{x+y}\, dxdy$. e) $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \ln(x^2+y^2+1)\, dydx$.

12. Find the volume of the given solid G.

a) G = solid in the first octant bounded by the coordinate planes, the cylinder $x^2 + y^2 = 4$ and the plane $y + z = 3$.
b) G = solid bounded above by the cylinder $x^2 + z^2 = 4$, below by the xy-plane and laterally by the cylinder $x^2 + y^2 = 4$. c) G = solid below the cone $z = \sqrt{x^2+y^2}$, inside the cylinder $x^2 + y^2 = 2y$, and above $z = 0$.
d) G = solid inside the sphere $r^2 + z^2 = 4$ and outside the cylinder $r = 2\cos\theta$.

13. Evaluate each integral using polar coordinates
a) $\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dydx$. b) $\int_0^a \int_0^{\sqrt{a^2-y^2}} \frac{dxdy}{(1+x^2+y^2)^{\frac{3}{2}}}$. c) $\int_0^1 \int_y^{\sqrt{y}} \sqrt{x^2+y^2}dxdy$.

14. Find the point B on the plane $x + 2y + 3z = 12$ that is closest to the point $A(1,2,-3)$. Find the distance between A and B.

15. A space probe in the shape of the ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters the Earth's atmosphere and its surface begins to heat. After one hour, the temperature at the point $(x,y,z$ on the probe's surface is $T(x,y,z) = 8x^2 + 4yz - 16z + 600$. Find the hottest point on the probe's surface.

16. Find the point on the sphere $x^2 + y^2 + z^2 = 4$ farthest from the point $D(1,-1,1)$.