MAC 2313 (Calculus III)
Test 2 Review: 13.1 to 13.8

1. Describe the domain of the function f in words. a) $f(x,y,z) = \ln(z^2 - x^2 - y^2)$, b) $f(x,y,z) = \cos^{-1}(x^2 + y^2 + z^2)$.

2. Sketch the largest region where f is continuous. a) $f(x,y) = \sqrt{x^2 + y^2 - 4}$, b) $f(x,y) = \sin^{-1}(y - x)$.

3. a) Find an equation for the level curve of the function f that passes through the point P. i) $f(x,y) = \int_x^y \frac{dt}{t^2+1}$, $P(-\sqrt{3}, \sqrt{3})$. ii) $f(x,y) = \sum_{n=0}^{\infty}(x/y)^n$, $P(1,2)$. b) Find an equation for the level surface of the function f that passes through the point P. i) $f(x,y,z) = \sum_{n=1}^{\infty} \frac{(-1)^n (xyz)^n}{n}$, $P(\sqrt{2}, 1, 1/\sqrt{2})$. ii) $f(x,y,z) = \int_x^y \frac{d\theta}{\sqrt{1-\theta^2}} + \int_{\sqrt{2}}^z \frac{dt}{t\sqrt{t^2-1}}$, $P(0, 1/2, 2)$.
c) Identify the level surfaces of $f(x,y,z) = \ln(x^2 + y^2 + z^2)$ for $k = -1,\ 0,\ 1$.

4. a) Let $f(x,y,z) = x^2 y^3 \sin(x^3 z^2)$. i) Find $f(y,z,x)$ and $f(z,x,y)$. ii) Find $f_x(x,y,z)$, $f_y(x,y,z)$ and $f_z(x,y,z)$.
b) Use implicit partial differentiation to find $\partial x/\partial y$ and $\partial x/\partial z$ if $xz + y\ln x - x^2 + 4 = 0$ defines x as a function of y and z. c) If $x = v\ln u$, $y = u\ln v$, use implicit partial differentiation to find u_x, v_x, u_y and v_y. If we set $z = \tan(2u - 3v)$, use the chain rule to find z_x and z_y. d) answer the same questions as in c) if $x = u^2 - v^2$, $y = u^2 - v$, and $z = u^2 + v^2$.

5. Evaluate each limit.

$$\text{a)} \lim_{(x,y,z)\to(-1,2,1)} \frac{xz^2}{\sqrt{x^2 + 2y^2 + 3z^2}},\ \text{b)} \lim_{(x,y)\to(1,1)} \frac{x^2 - 2xy + y^2}{x^2 - y^2},\ \text{c)} \lim_{(x,y)\to(-1,1)} \frac{2x^3 + 3x^2 y - 2xy^2 - 3y^3}{2x^2 + xy - y^2},$$

$$\text{d)} \lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2 + y^2 + z^2},\ \text{e)} \lim_{(x,y,z)\to(2,2,1)} \frac{\sin(2x - 5y + 6z)}{(2x - 5y + 6z)(y + z)},\ \text{f)} \lim_{(x,y)\to(0,0)} \frac{1 - \cos(x^2 + y^2)}{(x^2 + y^2)^2}.$$

6. Find an equation for the tangent plane and parametric equations for the normal line to the given surface at the given point. a) $2x^2 - 2y^2 + 4z^2 = 4$, $P(-1,1,1)$. b) $\frac{x + 2y}{2y + z} = 1$, $P(1, \sqrt{7}, -2)$.

7. Find parametric equations for the tangent line to the curve of intersection of the given surfaces at the given point. a) $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$, $P(\sqrt{2}, -\sqrt{2}, 4)$. b) $x^2 + y^2 = 2$ and $x + 2y + 3z = 6$, $P(1,1,1)$.

8. Let $f(x,y) = \begin{cases} \frac{xy}{x^2 + y^2} - 2x + 3y, & (x,y) \neq (0,0), \\ 0, & (x,y) = (0,0). \end{cases}$
a) Find $f_x(0,0)$, and $f_y(0,0)$. b) Show that f is not continuous at (0,0). c) Is f differentiable at $(0,0)$?

9. a) Write down the definition of " f is differentiable at (x_0, y_0)". b) Use the definition in a) to show that the function f given by $f(x,y) = 2x - 3xy$ is differentiable at the point $(1,-2)$.

10. a) Let $f(x,y,z) = x^3 e^{yz}$. i) Find the differential df. ii) Find the local linear approximation for f about $P(1,-1,-1)$, and use it to approximate $f(Q)$ with $Q(0.99, -1.01, -0.98)$. b) Answer the same questions for i) $f(x,y,z) = yz\ln(xy)$, $P(e,1,1)$ and $Q(2.72, 0.99, 1.01)$. ii) $f(x,y,z) = \tan^{-1}(xyz)$, $P(1,1,1)$ and $Q(0.98, 1.01, 0.99)$

11. Let $g(x,y,z) = xye^{x+yz}$. a) Find the gradient of g at $P(1,1,-1)$. b) Find a unit vector in the direction in which g decreases most rapidly at the point $Q(-1,1,1)$, and find the rate of change of g at Q in that direction. c) Find the directional derivative of g in the direction of the vector $\vec{a}=2\vec{i}-3\vec{j}+4\vec{k}$ at the point $A(1,-1,1)$.

12. Find all the critical points of f and classify them as points of local minimum, local maximum, or saddle points. a) $f(x,y) = xy + 2x - \ln(x^2 y)$, b) $f(x,y) = x^3 - y^3 - 2xy + 6$, c) $f(x,y) = 4xy + x^4 + y^4$, d) $f(x,y) = x^3 y^3$, e) $f(x,y) = 2y^2 x - x^2 y + 4xy$.

13. Find all the first partial derivatives of f if a) $f(x,y) = \log_x(y)$, b) $f(x,y) = \int_x^y g(t)\,dt$, c) $f(x,y,z) = yz\ln(xy)$, d) If z is defined implicitly by the relation $f(x,y,z) = 0$ as a differentiable function of x and y, where f is a differentiable function with $f_z(x,y,z) \neq 0$ for all allowable points (x,y,z), use implicit differentiation and the chain rule to find the partial derivatives z_x and z_y.

14. a) Find the point on the paraboloid $z = x^2 + y^2 + 10$ that is closest to the plane $x + 2y - z = 0$. b) Find three positive numbers whose sum is 48 and their product is as large as possible. c) Problems 44 and 48 in 13.8, p. 987.

15. Review the true/false problems in the text.