MAC 2313 (Calculus III)
Test 3, Thursday April 10, 2008

Name: **PID:**

Remember that no documents or calculators are allowed during the test. Be as precise as possible in your work; you must show all your work to deserve the full mark assigned to any question. Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University Administration. Good luck.

1. [20] a) Determine whether the lines: L1: $x = 2+5t, \quad y = 3+4t, \quad z = 1$ and L2: $x = 1+3t, \quad y = 7, \quad z = -1+t$ intersect. If so, find their point of intersection. Else, say whether the lines are parallel or skew.

b) Find the distance between the skew lines: L1: $x = 1 + t, \quad y = 2 - t, \quad z = 3 + t$ and L2: $x = 4 - t, \quad y = 3 + t, \quad z = -2 + t$.

2. [10] a) Let $\overrightarrow{u} = \overrightarrow{i} - \overrightarrow{j} + \overrightarrow{k}$ and $v = -2\overrightarrow{i} + \overrightarrow{k}$. Find $\overrightarrow{u} \times \overrightarrow{v}$. If θ denotes the angle between $\overrightarrow{u}$ and $\overrightarrow{v}$, find the exact value of $\sin(\theta)$.

3. [12] Let $F(x, y, z) = xy^2\overrightarrow{i} + yz^2\overrightarrow{j} + zx^2\overrightarrow{k}$. Find div F, and curl F.

4. [12] Evaluate the line integral $\displaystyle\int_{\mathcal{C}} \frac{\sin x}{y^2 + z^2} ds$ along the curve $\mathcal{C}$ parametrized by $\overrightarrow{r}(t) = t\,\overrightarrow{i} + \sin t\,\overrightarrow{j} + \cos t\,\overrightarrow{k}$, $0 \leq t \leq \pi$.

5. [15] Let $F(x, y) = (ye^x + 2e^x + y^2)\overrightarrow{i} + (e^x + 2xy)\overrightarrow{j}$. Show that F is conservative , and find a potential function φ for F. Evaluate the line integral $\int_{\mathcal{C}} (ye^x + 2e^x + y^2)\, dx + (e^x + 2xy)\, dy$ along the curve $\mathcal{C}$ parametrized by $r(t) = \cos^2(\pi t/4)\overrightarrow{i} + \sec^2(\pi t/4)\overrightarrow{j}, \quad 0 \leq t \leq 1$.

6. [7] Use Green's theorem to evaluate the line integral $\int_{\mathcal{C}} y^2\, dx - x^2\, dy$ where the curve $\mathcal{C}$ is the boundary of the region between $y = x^2$ and $y = x$ with counterclockwise orientation.

7. [10] Consider the surface $xz^2 - y^3 = 2$. a) Find an equation for the tangent plane to the surface at the point $Q(1, -1, 1)$. b) Find the parametric equations of the line normal to the surface at Q.

8. [14] a) Set up, but do not evaluate, the double integral (with the appropriate limits) equal to the surface integral $\int\int_\sigma xy^2\, dS$ by projecting σ on the xz-plane, where σ is the portion of the plane $2x + 3y + 4z = 12$ in the first octant.

b) Find the flux of the vector field $F(x, y, z) = x\,\overrightarrow{i} - \overrightarrow{k}$ across the surface σ, where σ is the portion of the plane $2x + y + z = 2$ in the first octant, oriented by upward unit normals. (You may just set up the double integral with the appropriate limits, and not evaluate it unless you have the time to do so).