MAC 2313 (Calculus III)
Test 3, Thursday November 30, 2006

Name: **PID:**

Remember that no documents or calculators are allowed during the test. Be as precise as possible in your work; you shall show all your work to deserve the full mark assigned to any question. Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University Administration. Good luck.

1. [26] Evaluate each integral.

a) $\int_0^1 \int_0^{\pi} \int_2^3 z^2(2r+1)\sin\theta \, dz d\theta dr =$

b) $\int_0^1 \int_1^2 (x^2 - y^2) \, dx dy =$

c) $\int_0^1 \int_e^{e^2} \ln y \, dy dx =$

d) $\int_0^2 \int_0^z \int_y^{\sqrt{y}} 2x \, dx dy dz =$

2. [10] Use polar coordinates to evaluate $\int_0^1 \int_0^x \sqrt{x^2+y^2}\, dydx$. ($\int \sec^n x\, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x\, dx$, $\int \sec x\, dx = \ln|\sec x + \tan x| + C$.)

3. [12] Let $F(x,y) = xy\,\overrightarrow{i} + yz\,\overrightarrow{j} + zx\,\overrightarrow{k}$. Find div F, and curl F.

4. [12] Evaluate the line integral $\int_C -y\, dx + x\, dy$ along the curve shown on the figure.

5. [15] Let $F(x,y) = (y\sin x - \cos y)\,\vec{i} + (x\sin y - \cos x)\,\vec{j}$. Show that F is conservative , and find a potential function φ for F. Evaluate the line integral $\displaystyle\int_{\mathcal{C}} (y\sin x - \cos y)\,dx + (x\sin y - \cos x)\,dy$ along the curve $\mathcal{C}$ parametrized by $r(t) = \tan(\pi t/4)\,\vec{i} + \tan^{-1} t\,\vec{j}, \quad 0 \le t \le 1$.

6. [7] Use Green's theorem to evaluate the line integral $\displaystyle\int_{\mathcal{C}} (2\tan^{-1}(y/x))\,dx + \ln(x^2+y^2)\,dy$ along the curve $\mathcal{C}$ parametrized by $r(t) = (3 + 2\cos t)\,\vec{i} + (1 + 3\sin t)\,\vec{j}, \quad 0 \le t \le 2\pi$.

7. [18] Let $f(x, y) = xy$. Use the Lagrange multipliers to find the maximum and minimum values of f subject to the constraint: $2x^2 + 3y^2 = 1$.

8. [Bonus, 10] State and prove the fundamental theorem of line integral.