MAC 2313 (Calculus III)
Test 3, Wednesday November 23, 2016

Name: **PID:**

Remember that no documents or calculators are allowed during the test. Be as precise as possible in your work; you shall show all your work to deserve the full mark assigned to any question. *You will not get any credit to any of the problems if you do not show your work.* **Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University Administration. Always do your best. Total=85 points.**

1. [13] Evaluate each integral: a) (polar coordinates) $\int_0^1 \int_0^{\sqrt{1-y^2}} \sin(\pi(x^2+y^2))\, dxdy =$

   b) (spherical coordinates) $\int_{-2}^2 \int_0^{\sqrt{4-y^2}} \int_0^{\sqrt{4-x^2-y^2}} \frac{1}{x^2+y^2+z^2+1}\, dzdxdy =$

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2. [12] a) Use cylindrical coordinates to evaluate the volume of the solid bounded above by the cone $z = 2 - \sqrt{x^2+y^2}$ and below by the paraboloid $z = x^2+y^2$. b) Write down the coordinates $(\bar{x}, \bar{y}, \bar{z})$ of the centroid of that solid as iterated triple integrals including all limits of integration, but do not evaluate these integrals.

3. [6] Evaluate the surface integral $\iint_\sigma z^2 dS$ if σ is the portion of the cone $z = 2\sqrt{x^2+y^2}$ between $z = 1$ and $z = 2$.

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4. [8] Find the mass of the lamina represented by the parametric surface $\vec{r}(u,v) = u\cos v\,\vec{i} + u\,\vec{j} + u\sin v\,\vec{k}$ if $1 \le u \le 2$ and $0 \le v \le \pi$, and its density is $\delta(x,y,z) = \sqrt{x^2+y^2+z^2}$.

5. [10] Reverse the order of integration and evaluate the integral:

$$\int_0^2 \int_r^2 r\sqrt{r^2+z^2}\,dzdr =$$

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6. [10] a) Find div$\mathbf{F}(x,y,z)$ and curl$\mathbf{F}(x,y,z)$ if $\mathbf{F}(x,y,z) = y^2x\,\vec{i} - z^2y\,\vec{j} + x^2z\,\vec{k}$.

b) Find the Jacobian $\dfrac{\partial(x,y)}{\partial(u,v)}$ of the transformation $u = 2x - y^2$, $v = xy$.

7. [15] Let $F(x,y) = (2x\sin y + y^3 e^x)\vec{i} + (x^2\cos y + 3y^2 e^x)\vec{j}$. a) Show that F is conservative. b) Find a potential function φ for F. c) Evaluate the line integral $\int_{\mathcal{C}} (2x\sin y + y^3 e^x)\, dx + (x^2\cos y + 3y^2 e^x)\, dy$ along the curve $\mathcal{C}$ parametrized by $\vec{r}(t) = \ln(1+t)\vec{i} + \tan^{-1} t\,\vec{j}, \quad 0 \le t \le 1$.

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8. [11] a) State Green's Theorem.

b) Use Green's Theorem to evaluate the line integral $K = \oint_{\mathcal{C}} (2xy + e^{\cos x} - 3y)\, dx + (x^2 + \ln(1+\sin^2 y))\, dy$, where $\mathcal{C}$ is the boundary of the region in the first quadrant enclosed by the curves $y = 4x$ and $y = x^3$ with a counterclockwise orientation.