

MAP 2302 (Differential Equations)
Test 3, Friday April 13, 2018

Name:

PID:

Remember that no documents or calculators are allowed during the test. You shall show all your work to deserve the full mark assigned to any question. Always do your best.

$$\mathcal{L}(t^n)(s) = n!/s^{n+1}, \mathcal{L}(\sin(bt))(s) = b/(s^2 + b^2), \mathcal{L}(\cos(bt))(s) = s/(s^2 + b^2), \mathcal{L}(e^{at})(s) = 1/s - a.$$

1. [20] We denote the inverse Laplace transform by \mathcal{L}^{-1} . Find $\mathcal{L}^{-1}\left(\frac{2s^2+s+8}{(s^2-s-12)(s^2+4)}\right)$

2. [20] Use Laplace transform to solve the initial-value problem: $\begin{cases} y'' - y = 6e^{-2t} + 4 \\ y(0) = 0; y'(0) = 0. \end{cases}$

3. [20] Find the Laplace transform $\mathcal{L}h$ if $h(t) = \begin{cases} 5, & 0 \leq t < 4, \\ -3, & 4 < t < 6 \\ t, & t > 6. \end{cases}$

4. [20] Find two linearly independent power series solutions of the differential equation: $y'' + xy' - 3y = 0$. Include in each series the powers of x up to x^4 . Write down the general solution of the differential equation.