MAC 2311 (Calculus I)
Test 3 Review- Spring 2015

1. Let l be the length of a diagonal in a rectangle whose sides have lengths x and y, and assume that x and y vary with time. a) How are dl/dt, dx/dt and dy/dt related? b) If x increases at a rate of 6 in$/s$ and y decreases at a rate of 3 in$/s$, how fast is l changing when $x = 3$ft and $y = 4$ft? Is the diagonal increasing or decreasing at that instant?

2. A 17ft ladder is leaning against a wall. If the bottom of the ladder is pulled along the ground away from the wall at a constant rate of 5ft$/s$, how fast will the top of the ladder be moving down the wall when it is 8ft above the ground?

3. A point P is moving along the curve $y = \sqrt{x^3 + 17}$. When P is at (2,5), y is increasing at a rate of 2 units$/s$. How fast is x changing?

4. An aircraft is climbing at a 30° angle to the horizontal. How fast is the aircraft gaining altitude if its speed is 500 mi$/h$?

5. A steel cube with 1-inch side is coated with 0.01 inch of copper. a) What is the volume of cooper in the coating? b) Use differentials to estimate the volume of copper in the coating.

6. Find the differential dy if: a) $y = \frac{1-x^3}{2-x}$, b) $y = \cos(sinx)$, c) $y = \tan^{-1}(e^{\sin x})$.

7. Use the differential dy to approximate Δy when x changes as indicated. a) $y = \frac{x}{x^2+1}$; from $x = 2$ to $x = 1.96$. b) $y = x\sqrt{x^2 + 3}$; from $x = 1$ to $x = 1.02$.

8. Use an appropriate local linear approximation to estimate the given value. a) $\sqrt{80.82}$, b) ln(1.01), c) tan(44°), d) sin(31°).

9. Evaluate each limit.

a) $\lim\limits_{x\to 0^+} \dfrac{1 - \ln x}{e^{\frac{1}{x}}}$, b) $\lim\limits_{x\to +\infty} x^2(1 - \cos(1/x))$, c) $\lim\limits_{x\to 0} x^2(1 - \cos(1/x))$, d) a, b are constants with $b > 0$. $\lim\limits_{x\to -\infty} (1 + \dfrac{a}{x})^{bx}$, e) $\lim\limits_{x\to \frac{\pi}{2}^-} (\tan x)^{(\frac{\pi}{2}-x)}$, f) $\lim\limits_{x\to 0} \dfrac{x - \tan x}{x^3}$, g) $\lim\limits_{x\to 0^-} (\dfrac{1}{x} - \cot x)$,
h) $\lim\limits_{x\to +\infty} \dfrac{x}{\sqrt{x^2 + 1}}$.

10. Mark each statement true or false.

a) If f is decreasing on [0,2], then $f(0) > f(1) > f(2)$.
b) If $f'(1) > 0$, then f is increasing on [0,2].
c) If f' is increasing on [0,1] and f' is decreasing on [1,2], then $(1, f(1))$ is an inflection point of f.
d) If f has a relative maximum at $x = 1$, then $f(1) \geq (2)$.
e) If f has a relative maximum at $x = -1$, then $x = -1$ is a critical point of f.
f) If $f''(1) > 0$, then f has a relative minimum at $x = 1$.
g) If $f'(-2) = 0$, then f has a relative maximum or a relative minimum at $x = -2$.
h) If $f'(-3) = 0$ and $f''(-3) < 0$, then f has a relative maximum at $x = -3$.
i) If $f''(5) = 0$, then the point $(5, f(5))$ is an inflection point of f.
j) If $f'(3)$ does not exist, then $x = 3$ is a critical point of f.

11. Find the intervals of increase, decrease, concavity, and the inflection points of f.

a) $f(x) = x^{\frac{4}{3}} - x^{\frac{1}{4}}$, b) $f(x) = x^4 - 5x^3 + 9x^2$, c) $f(x) = x^3 \ln x$, d) $f(x) = xe^{-x^2}$,
e) $f(x) = \tan^{-1}(1 - x^2)$.

12. Find and classify all the critical points of f as points of local maximum, local minimum or neither.

a) $f(x) = x^4 - 12x^3 + 1$, b) $f(x) = x^2(x + 1)^{\frac{2}{3}}$, c) $f(x) = \sqrt{3} - \sin(2x)$, $0 \leq x \leq 2\pi$,

d) $f(x) = x^2 e^{2x}$, e) $f(x) = |3x - x^2|$.

13. Find all the intercepts, asymptotes, intervals of increase, decrease, concavity, critical points, inflection points of f.

a) $f(x) = \frac{x^3-4x-8}{x+2}$, b) $f(x) = x^{\frac{2}{3}} e^x$, c) $f(x) = \sin^2 x - \cos x$, $-\pi \leq x \leq \pi$.

14. Find the absolute maximum and minimum values of f and state where they occur.

a) $f(x) = 2x^3 + 3x^2 - 12x$, $-3 \leq x \leq 2$, b) $f(x) = x - 2\sin x$, $0 \leq x \leq \pi$,
c) $f(x) = \frac{x-2}{x+1}$, $-1 < x \leq 5$.

15. Let $f(x) = x^2 + px + q$. Find all values of p and q such that $f(1) = 3$ is an extreme value of f on the interval [0,2]. Is this value a maximum or a minimum?

16. A rectangular field with an area of 3200 ft^2 is to be fenced off. Two opposite sides will use fencing costing \$1 per foot and the remaining sides will use fencing costing \$2 per foot. Find the dimensions of the field with least cost.

17. A church window consisting of a rectangle topped by a semi-circle is to have a perimeter p. Find the radius of the semi-circle if the area of the window is to be a maximum.

18. The shoreline of Circle Lake is a circle with diameter 2 mi. Nancy's training routine begins at point E on the Eastern shore of the lake. She jogs along the north shore to a point P, and then swims the straight line distance,if any, from P to the point W diametrically opposite E. Nancy swims at a rate of 2 mi/h and jogs a 8 mi/h. How far should Nancy jog in order to complete her training routine in a) the least amount of time? b) the greatest amount of time?