

CALCULUS I EXAM 2**NAME** _____

(1) Find the derivative $\frac{dy}{dx}$. You need not simplify your answer.

(a) (7 points) $y = \left(\frac{1}{x} - 5\right) (\sqrt{x} + \sec x)$

(b) (7 points) $y = \sqrt{\cos(e^{3x})}$

(c) (7 points) $y = \frac{2^x \sin x}{\ln x}$

(d) (7 points) $y = (\tan^{-1} x) \log_2(1 - x^3)$

(2) (9 points) Let $xy^2 + e^x = \cos(y^3)$. Find $\frac{dy}{dx}$ by **implicit differentiation**.

(3) (10 points) Let $y + \sin y = x$. Find the second derivative $\frac{d^2y}{dx^2}$ by **implicit differentiation**.

(4) (9 points)

(a) Find the local linear approximation of $f(x) = e^x$ at $x_0 = 0$.

(b) Use the local linear approximation obtained in (a) to approximate $e^{0.1}$.

(5) (6 points) Determine whether the function $f(x) = x^5 + x^3 + 2x - 17$ is one-to-one by examining the sign of $f'(x)$.

(6) (9 points) Let $f(x) = x^3 + \tan x$, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Find the derivative of the inverse function f^{-1} .

(7) (10 points) Let

$$y = \frac{x^4 \sqrt{x^2 + 1} \csc^3 x}{x^3 + 7}$$

Find the derivative $\frac{dy}{dx}$ **using logarithmic differentiation**. No credit will be given if taking the derivative directly.

(8) (9 points) A 10 ft ladder is leaning against a wall. If the bottom of the ladder is pulled along the ground away from the wall at a constant rate of 5 ft/s, how fast will the top of the ladder be moving down the wall when it is 8 ft above the ground?

(9) (10 points) A conical water tank with vertex down has a radius of 4 ft at the top and is 12 ft high. If water flows into the tank at a rate of $5 \text{ ft}^3/\text{min}$, how fast is the depth of the water increasing when the water is 10 ft deep?