CALCULUS I EXAM 3 **NAME** ______________________________

(1) Find the limit.

(a) (6 points) $\lim_{x \to 0} \dfrac{e^x - 1}{\sin x}$

(b) (6 points) $\lim_{x \to +\infty} \dfrac{\ln x}{x}$

(c) (7 points) $\lim_{x \to 0} \left(\dfrac{1}{x} - \dfrac{1}{e^x - 1} \right)$

(2) (5 points) Let $f(x) = \dfrac{5x^2 + 1}{x}$. Find the horizontal, vertical, and oblique asymptotes, if any.

(3) Let $f(x) = x^3 - 3x^2$. Answer the following questions.

(a) (2 point) x-intercept(s): ________________________________

(b) (2 points) Interval(s) on which f is increasing: ________________________________

(c) (2 points) Relative maxima: __________________ Relative minima: __________________

(d) (2 points) Interval(s) on which f is concave up: ________________________________

(e) (2 points) Inflection point(s): ________________________________

(f) (3 points) Sketch the graph and label the coordinates of the intercepts, relative extrema, and inflection point(s).

(4) Let $f(x) = \dfrac{8}{x^2 - 4}$. Answer the following questions. Need NOT sketch the graph.

(a) (2 point) y-intercept: ____________________

(b) (2 point) Symmetric about the y-axis? about the origin? ____________________

(c) (2 point) Horizontal asymptote: ____________________

(d) (2 point) Vertical asymptotes: ____________________

(e) (3 point) Interval(s) on which f is increasing: ____________________

(5) (7 points) Find the **absolute** maximum and minimum values of $f(x) = 4x^3 + x^4$, if any, on $(-\infty, +\infty)$, and state where those values occur.

(6) (9 points) Find the **absolute** maximum and minimum values of $f(x) = (x^2 - 2x)^{\frac{2}{3}}$ on the closed interval $[-2, 3]$, and state where those values occur.

(7) Find the antiderivative.

(a) (6 points) $\displaystyle\int \frac{x^3 + 5x^2 + 1}{x^2}\, dx$

(b) (6 points) $\displaystyle\int \sec^2 x + e^x + \frac{1}{x}\, dx$

(c) (6 points) $\int x^2 \cos(x^3 + 1)\, dx$

(8) (8 points) Verify that the hypotheses of the Mean-Value Theorem are satisfied for $f(x) = x^3 + x - 4$ on the interval $[-2, 2]$, and find all values of c in $(-2, 2)$ that satisfy the conclusion of the theorem.

(9) (10 points) A closed cylindrical can is to hold 50 cm^3 of liquid. Find the height and radius that minimize the amount of material needed to manufacture the can.