CALCULUS II EXAM 1 (2/9/17) **NAME** ____________________

You have 100 minutes to finish the exam. Please show all your work for full credits.

Formula: $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$; $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$; $\sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$

1. (3 points) Express $a_1 - a_2 + a_3 - a_4 + a_5$ in sigma notation.

2. (12 points) Use $A = \lim_{n\to+\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$ with x_k^* as the *right* endpoint of each subinterval to find the area under the curve $f(x) = 9 - x^2$ over the interval $[1, 3]$. (*No credit will be given for using other methods.*)

3. (8 points) Find the total area between the curve $y = 4 - x^2$ and the interval $[0, 3]$ on the x-axis.

4. (8 points) Evaluate $\int_1^4 f(x)dx$ if f is a piecewise defined function

$$f(x) = \begin{cases} 1/x, & x < 2 \\ 1 - x, & x \geq 2 \end{cases}$$

5. (9 points) Let $F(x) = \int_1^x \tan^{-1} t\, dt$.

   Find (a) $F(1)$ (b) $F'(1)$ (c) $F''(1)$

6. (8 points) Let $f(x) = x^2$. Find all values of x^* in the interval $[-3, 3]$ that satisfy the formula in the Mean Value Theorem for integrals.

7. (8 points) Find the average value of the function $f(x) = \sec x \tan x$ over the interval $\left[0, \frac{\pi}{3}\right]$.

8. Evaluate the integral.

   (a) (8 points) $\displaystyle\int_0^{\frac{\pi}{2}} \sin x \cos x \, dx$

   (b) (8 points) $\displaystyle\int_1^5 \frac{x}{\sqrt{x-1}} \, dx$

(c) (8 points) $\int_{-1}^{2} xe^{-x^2}\,dx$

9. Sketch the region enclosed by the curves and find the area.

(a) (10 points) $y = x^2, \quad y = x + 2$

(b) (10 points) $y = x, \quad y = 2x, \quad y = -x + 6$