CALCULUS I EXAM 2 **NAME** ______________________________

(1) Find the derivative $\frac{dy}{dx}$. Do not simplify your answer.

(a) $y = \left(\frac{1}{x} + \tan^{-1} x\right) \sin^3 x$

(b) $y = \sqrt{\ln|x| + e^{3x}}$

(c) $y = \cos^3(\sec x)$

(d) $y = \dfrac{2^x \sin^{-1}(x^2)}{\csc(x^3)}$

(e) $y = 5^{\tan x} \log_5(2 - x^2)$

(2) Let $y = x\sin(4x) + \cos^2 x$. Find the **second derivative** $\dfrac{d^2y}{dx^2}$.

(3) Let $x^2y^3 + \cot x = \sin(y^2)$. Find $\dfrac{dy}{dx}$ by **implicit differentiation**.

(4) Let $x^3y^3 + 5 = 0$. Find the second derivative $\dfrac{d^2y}{dx^2}$ by **implicit differentiation**.

(5) (a) Find the local linear approximation of $f(x) = \sqrt{x}$ at $x_0 = 9$.
(b) Use the local linear approximation obtained in (a) to approximate $\sqrt{8.8}$.

(6) Determine whether the function $f(x) = x^3 - 2x^2 + 7$ is one-to-one by examining the sign of $f'(x)$.

(7) Let $f(x) = x^2 + e^{5x}$. Find the derivative of the inverse function f^{-1}.

(8) Let

$$y = \frac{x^4\sqrt{1-x^3}\sin^2 x}{\tan^3 x}$$

Find the derivative $\dfrac{dy}{dx}$ **using logarithmic differentiation**. No credits for taking the derivative directly.

(9) A rocket, rising vertically, is tracked by a radar station that is on the ground 3 miles from the launchpad. How fast is the rocket rising when it 4 miles high and its distance from the radar station is increasing at a rate of 2000 miles/hour?

(10) A conical water tank with vertex down has a radius of 5 ft at the top and is 10 ft high. If water flows into the tank at a rate of 9 ft^3/min, how fast is the depth of the water increasing when the water is 3 ft deep?