CALCULUS I EXAM 3 **NAME** ______________________________

(1) Find the limit.

(a) (6 points) $\lim_{x \to 0} \frac{e^{2x} - 1}{\sin x}$

(b) (6 points) $\lim_{x \to +\infty} \frac{x^2 + 8x + 7}{e^x + 2x - 50}$

(c) (6 points) $\lim_{x \to +\infty} x \sin \frac{\pi}{x}$

(d) (6 points) $\lim_{x \to 0} \left(\csc x - \frac{1}{x} \right)$

(2) Let $f(x) = 4x^3 - x^4$. Answer the following questions.

(a) (1 point) x-intercept(s): ______________________________

(b) (1 point) Critical points: ______________________________

(c) (1 point) Interval(s) on which f is increasing: ______________________________

(d) (1 point) Relative maxima: ______________________________

(e) (1 point) Relative minima: ______________________________

(f) (1 point) Interval(s) on which f is concave up: ______________________________

(g) (1 point) Inflection point(s): ______________________________

(h) (3 points) Sketch the graph and label the coordinates of the intercepts, relative extrema, and inflection point(s).

(3) (5 points) Let $f(x) = \dfrac{3x^2 - 2}{x}$. Find the horizontal, vertical, and oblique asymptotes, if any.

(4) Let $f(x) = \dfrac{3-x}{x-2}$. Answer the following questions.

(a) (1 point) x-intercept(s): ______________________________

(b) (1 point) y-intercept: ______________________________

(c) (1 point) Determine whether there is symmetry about the y-axis or the origin. ______________

(d) (1 point) Horizontal asymptote: ______________________________

(e) (1 point) Vertical asymptote: ______________________________

(f) (1 point) Interval(s) on which f is decreasing: ______________________________

(g) (1 point) Interval(s) on which f is concave up: ______________________________

(h) (3 points) Sketch the graph and label the coordinates of the intercepts and asymptotes.

(5) (7 points) Find the **absolute** maximum and minimum values of $f(x) = 4x^3 - 3x^4$, if any, on $(-\infty, +\infty)$, and state where those values occur.

(6) (8 points) Find the **absolute** maximum and minimum values of $f(x) = (x^2 + x)^{\frac{2}{3}}$ on the closed interval $[-2, 3]$, and state where those values occur.

(7) Find the antiderivative.

(a) (6 points) $\displaystyle\int \sqrt{x} + \frac{x^2+1}{x}\, dx$

(b) (6 points) $\displaystyle\int \sec x(\sec x + \tan x)\, dx$

(c) (6 points) $\displaystyle\int \frac{x}{(x^2+1)^4}\,dx$

(8) (8 points) Verify that the hypotheses of the Mean-Value Theorem are satisfied for $f(x) = x - \dfrac{1}{x}$ on the interval $[3, 4]$, and find all values of c in $(3, 4)$ that satisfy the conclusion of the theorem.

(9) (10 points) A closed cylindrical can is to hold 100 cm^3 of liquid. Find the height and radius that minimize the amount of material needed to manufacture the can.