REVIEW PROBLEMS FOR THE LECTURE ON 12/8/2017

1. Find the limits.

(a) $\lim_{x\to 2} \dfrac{2x^2-3x+2}{x^2+4x+4}$

(b) $\lim_{x\to 2} \dfrac{2x^2-3x+2}{x^2-4x+4}$

(c) $\lim_{x\to +\infty} \dfrac{2x^2-3x+2}{x^2-4x+4}$

(d) $\lim_{x\to 3} \dfrac{9-x^2}{3-x}$

(e) $\lim_{x\to 6^+} \dfrac{x-3}{x^2-x-30}$

(f) $\lim_{x\to +\infty} \dfrac{\sqrt{2x^2-3}}{x-2}$

(g) $\lim_{x\to \infty} \dfrac{e^{2x}}{x^2}$

(h) $\lim_{x\to 0} \dfrac{\tan(4x)}{\sin(3x)}$

(i) $\lim_{x\to 0} \dfrac{x^2}{1-\cos^2 x}$

(j) $\lim_{x\to\infty} xe^{-x}$

(k) $\lim_{x\to\infty} x\sin\left(\frac{\pi}{x}\right)$

(l) $\lim_{x\to 0^+} \left(\frac{1}{x} - \csc x\right)$

2. Find all values of x, if any, at which f is not continuous. Classify each discontinuity: is it a removable discontinuity, jump discontinuity, or vertical asymptote?

(a) $f(x) = \dfrac{x}{x^2-4}$

(b) $f(x) = \dfrac{x}{x^2+4}$

(c) $f(x) = \tan x$

(d) $f(x) = \begin{cases} 2 + \dfrac{3}{x} & \text{if } x \le 1 \\ 2x - 1 & \text{if } x > 1 \end{cases}$

(e) $f(x) = \dfrac{x^2-9}{x^2-2x-3}$

3. Find the derivative $\dfrac{dy}{dx}$ of the functions **by using a limiting process.** (No credit will be given if using other methods.)

   (a) $y = \dfrac{1}{x}$

   (b) $y = \sqrt{x}$

4. Find the derivative $\dfrac{dy}{dx}$.

   (a) $y = \dfrac{3x-1}{x^2+7}$

   (b) $y = e^{x^2}\sin(5x)$

   (c) $y = \left(\tan^{-1} x\right)\ln(3x+1)$

   (d) $y = \cos^3(\tan 5x)$

   (e) $y = \sin(\sqrt{x}) + \sqrt{\sin x}$

   (f) $y = x^5 + 5^x + e^{3x} + \ln(3x) - \ln 7$

5. Consider the equation $y^5 - 2xy + 3x^2 = 9$. Use **implicit differentiation** to find $\dfrac{dy}{dx}$.

6. Use **logarithmic differentiation** to find the derivative of $y = \dfrac{(x^2+6)3^{2x-1}}{\sqrt[5]{2x-3}}$.

7. Let $f(x) = x^5 + e^{-2x}$. Find the derivative of the inverse function f^{-1}.

8. Given a parametric curve $x = t + \cos t$, $y = 1 + \sin t$. Find $\dfrac{dy}{dx}$ without eliminating the parameter.

9. Find the antiderivatives.

   (a) $\displaystyle\int \left(x^{-3} - 3\sqrt[4]{x} + 8\sin x\right) dx$

   (b) $\displaystyle\int \left(2^x + \frac{1}{2x}\right) dx$

   (c) $\displaystyle\int \left(\frac{1}{\sqrt{1-x^2}} - 4\sec x \tan x\right) dx$

(d) $\displaystyle\int \frac{x^5 + 2x^3 - 1}{x^4}\, dx$

(e) $\displaystyle\int 2x(x^2 + 1)^{23}\, dx$

(f) $\displaystyle\int x^3\sqrt{x^4 + 1}\, dx$

(g) $\displaystyle\int x^2 e^{x^3}\, dx$

(h) $\displaystyle\int \frac{1}{1 - 3x}\, dx$

(i) $\displaystyle\int \cos^3 x \sin x\, dx$