Test of Hypothesis for population mean μ

① set up H_0, H_a. :

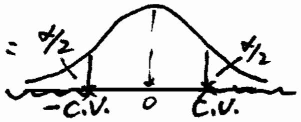
$H_0: \mu = \mu_0$, $H_a: \mu \neq \mu_0$ (two-tailed) or $H_0: \mu = \mu_0$, $H_a: \mu < \mu_0$ (lower-tailed) or $H_0: \mu = \mu_0$, $H_a: \mu > \mu_0$ (upper-tailed)

② specify the level of significance : α

③ calulate the test statistic value : (T.S.)

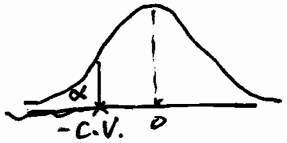
- large sample ($n \geq 30$)
  - σ known: $Z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$
  - σ unknown: $Z = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$
- small sample ($n < 30$): $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

④ rejection region :



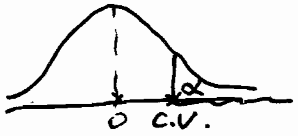
(z-test): $z < -z_{\alpha/2}$ or $z > z_{\alpha/2}$

(t-test): $t < -t_{\alpha/2}$ or $t > t_{\alpha/2}$



$z < -z_\alpha$

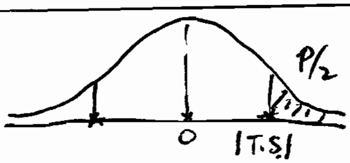
$t < -t_\alpha$



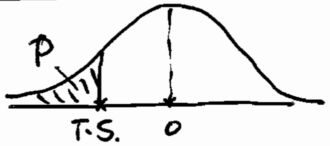
$z > z_\alpha$

$t > t_\alpha$

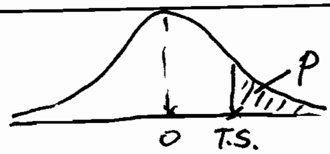
*④ find p-value :



(z-test): p-value $= 2 \cdot P(z \geq |z_0|)$



p-value $= P(z < z_0)$



p-value $= P(z > z_0)$

⑤ make decision and conclusion. :

if test statistic value fall in R.R., or if p-value smaller than α, reject H_0, and conclude there is enough evidence H_a is true.

otherwise, fail to reject H_0, and conclude there is not enough evidence H_a is true.

Test of hypothesis for population proportion p

① set up H_0, H_a. :

two-tailed	lower-tailed	upper-tailed
$H_0: p = p_0$	$H_0: p = p_0$	$H_0: p = p_0$
$H_a: p \neq p_0$	$H_a: p < p_0$	$H_a: p > p_0$
(two-tailed)	or (lower-tailed)	or (upper-tailed)

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② specify level of significance : α

③ calculate test statistic value : large sample ($np_0 \geq 15$ and $nq_0 \geq 15$) : $Z = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$

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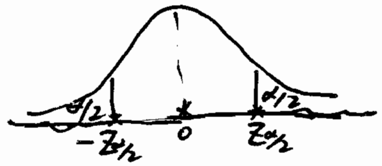
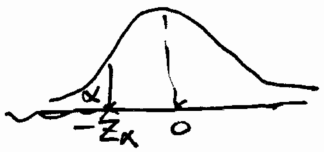
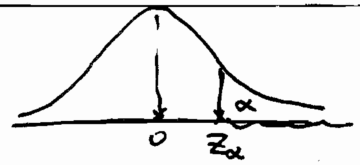
④ rejection region :



$Z < -Z_{\alpha/2}$ or $Z > Z_{\alpha/2}$



$Z < -Z_\alpha$



$Z > Z_\alpha$

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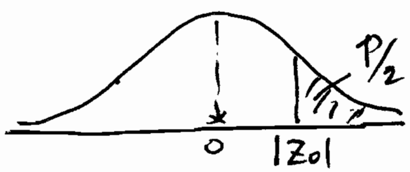
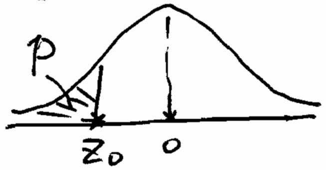
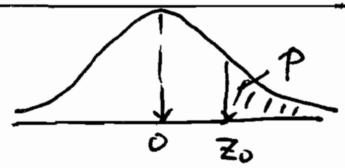
* ④ P-value :



P-value $= 2 \cdot P(Z \geq |Z_0|)$



P-value $= P(Z < Z_0)$



P-value $= P(Z > Z_0)$

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⑤ make decision and conclusion. :

if ~~test~~ test statistic value falls in R.R, or if P-value $< \alpha$, reject H_0, and conclude there is enough evidence H_a is true.

otherwise, fail to reject H_0, and conclude there is insufficient evidence H_a is true.

Estimation

Estimation of population mean μ

- point est. : sample mean $\bar{x}$
- confidence interval est. :
  - large sample ($n \geq 30$)
    - σ known : $\bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$
    - σ unknown : $\bar{x} \pm Z_{\alpha/2} \frac{s}{\sqrt{n}}$
  - small sample ($n < 30$) : $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$
- minimum sample size required : $n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2$

Estimation of population proportion p

- point est : sample proportion $\hat{p} = \frac{x}{n}$
- confidence interval est. : Large sample ($n\hat{p} \geq 15$ and $n\hat{q} \geq 15$) : $\hat{p} \pm Z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$
- minimum sample size required : $n = \hat{p}\hat{q} \left(\frac{Z_{\alpha/2}}{E}\right)^2$