Chapter 4 Discrete Random variables

A ________________ is a variable that assumes numerical values associated with the random outcomes of an experiment, where only one numerical value is assigned to each sample point.

Example1: define random variable x = the # of heads observed when tossing two coins,

X can be _________________.

Random variable	sample points
x = ___,	{TT}
x = ___,	{HT, TH}
x = ___,	{HH}

Example2: define random variable X= the number of boys in a family with three children.

X can be _______________.

Random variable	sample points
x = ___, (no boy)	{GGG}
x = ___, (one boy)	{BGG, GBG, GGB}
x = ___, (two boys)	{BBG, BGB, GBB}
x = ___, (three boys)	{BBB}

Example3. define random variable X = the sum of the two dice when tossing two dice,

X can be _________________________________

You can list the corresponding sample points to each value of X.

4.1 Two Types of Random Variables

- Random variables that can assume a _________number of values are called _________.
- Random variables that can assume values corresponding to _____ of the points contained in an ___________are called _______________.

The following are examples of discrete random variables:

1. The number of seizures an epileptic patient has in a given week: $x = 0, 1, 2, \ldots$
2. The number of voters in a sample of 500 who favor impeachment of the president: $x = 0, 1, 2, \ldots, 500$
3. The number of students applying to medical schools this year: $x = 0, 1, 2, \ldots$
4. The change received for paying a bill: $x = 1¢, 2¢, 3¢, \ldots, \$1, \ldots$
5. The number of customers waiting to be served in a restaurant at a particular time: $x = 0, 1, 2, \ldots$

Note that several of the examples of discrete random variables begin with the words *The number of* . . . . This wording is very common, since the discrete random variables most frequently observed are counts. The following are examples of continuous random variables:

1. The length of time (in seconds) between arrivals at a hospital clinic: $0 \le x \le \infty$ (infinity)
2. The length of time (in minutes) it takes a student to complete a one-hour exam: $0 \le x \le 60$
3. The amount (in ounces) of carbonated beverage loaded into a 12-ounce can in a can-filling operation: $0 \le x \le 12$
4. The depth (in feet) at which a successful oil-drilling venture first strikes oil: $0 \le x \le c$, where c is the maximum depth obtainable
5. The weight (in pounds) of a food item bought in a supermarket: $0 \le x \le 500$ [*Note:* Theoretically, there is no upper limit on x, but it is unlikely that it would exceed 500 pounds.]

4.2 Probability Distribution for Discrete Random Variables

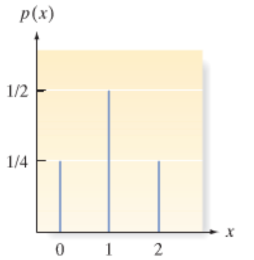
- The _______________ of a discrete random variable is a ____, _____, or __________ that specifies the probability associated with each possible value that the random variable can assume.

Example1: define random variable x = the # of heads observed when tossing two coins, describe the probability distribution for X.

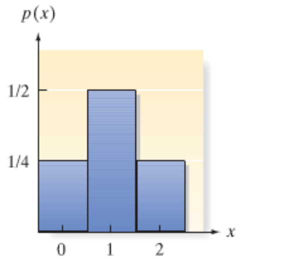
X can be 0, 1, 2.

Random variable	sample points
X = 0, (no heads)	{TT}
X = 1, (one head)	{HT, TH}
X = 2, (two heads)	{HH}

- Probability distribution can be given by graph:



a. Point representation of $p(x)$



b. Histogram representation of $p(x)$

- Probability distribution can be given by table:

- Probability distribution can be given by formula:

Example2: define random variable X= the number of boys in a family with three children, describe the probability distribution for X.

X can be 0, 1, 2, 3.

Random variable	sample points
X = 0 (no boy)	{GGG}
X = 1 (one boy)	{BGG, GBG, GGB}
X = 2 (two boys)	{BBG, BGB, GBB}
X = 3 (three boys)	{BBB}

- Probability distribution given by table:

x				
P(X= x)				

- Probability distribution given by Graph:

- Probability distribution given by formula:

- **Two requirements must be satisfied by all probability distributions for discrete random variable:**

Example: The following is the probability distribution of random variable X.

x	10	20	30	40
P(x)	.15	.20	?	.25

1. What are the possible values for the random variable X?
2. What is the probability of x = 30?
3. What is the probability that x is at most 30?
4. What is the probability that x is greater than 20?
5. What is the probability that x = 25?

4.3 Expected values of discrete random variables

- **Mean or Expected value of a discrete R.V.,**

Example1: The following is the probability distribution of random variable X.

x	10	20	30	40
P(x)	.15	.20	0.40	.25

Find the mean (expected value) of random variable x.

Example2: A local bakery has determined a probability distribution for the number of cheesecakes it sells in a given day. The distribution is as follows:

Number sold in a day	0	5	10	15	20
Prob (Number sold)	0.21	0.15	0.06	0.07	0.51

1. Find the number of cheesecakes that this local bakery expects to sell in a day.

2. What is the probability that the number of cheesecakes it sells in a given day is at least 10?

Example3: A dice game involves rolling three dice and betting on one of the six numbers that are on the dice. The game costs $8 to play, and you win if the number you bet appears on any of the dice. The distribution for the outcomes of the game (including the profit) is shown below:

Number of dice with your number	Profit	Probability
0	–$8	125/216
1	$8	75/216
2	$10	15/216
3	$24	1/216

Find your expected profit from playing this game.

Example 4: At a raffle, 1500 tickets are sold at $2 each for three prizes of $500, $300 and $200. You buy one ticket. What is the expected value of your gain?

Problem Suppose you work for an insurance company and you sell a $10,000 one-year term insurance policy at an annual premium of $290. Actuarial tables show that the probability of death during the next year for a person of your customer's age, sex, health, etc., is .001. What is the expected gain (amount of money made by the company) for a policy of this type?

- **The variance of a random variable:**

____

- **The standard deviation of a random variable:**

____

Chebyshev's Rule and Empirical Rule for a Discrete Random Variable

Let x be a discrete random variable with probability distribution $p(x)$, mean μ, and standard deviation σ. Then, depending on the shape of $p(x)$, the following probability statements can be made:

	Chebyshev's Rule	Empirical Rule
	Applies to any probability distribution (see Figure 4.5a)	Applies to probability distributions that are mound shaped and symmetric (see Figure 4.5b)
$P(\mu - \sigma < x < \mu + \sigma)$	≥ 0	$\approx .68$
$P(\mu - 2\sigma < x < \mu + 2\sigma)$	$\geq 3/4$	$\approx .95$
$P(\mu - 3\sigma < x < \mu + 3\sigma)$	$\geq 8/9$	≈ 1.00

Example1: define random variable x = the # of heads observed when tossing two coins, The probability distribution is given in the following table.

x	P(X = x)
0	0.25
1	0.50
2	0.25

1. Find the expected number of heads (mean number of heads) we wish to observe.

2. Find the standard deviation of the number of heads.

3. Find the probability that the number of heads fall in two standard deviations within the mean.

Problem Medical research has shown that a certain type of chemotherapy is successful 70% of the time when used to treat skin cancer. Suppose five skin cancer patients are treated with this type of chemotherapy, and let x equal the number of successful cures out of the five. The probability distribution for the number x of successful cures out of five is given in the following table:

x	0	1	2	3	4	5
$p(x)$	.002	.029	.132	.309	.360	.168

a. Find $\mu = E(x)$. Interpret the result.

b. Find $\sigma = \sqrt{E[(x - \mu)^2]}$. Interpret the result.

c. Graph $p(x)$. Locate μ and the interval $\mu \pm 2\sigma$ on the graph. Use either Chebyshev's rule or the empirical rule to approximate the probability that x falls into this interval. Compare your result with the actual probability.

d. Would you expect to observe fewer than two successful cures out of five?

e. What is the probability there would be at least three successful cures out of five patients?

4.4 The Binomial Distribution

- **Characteristics of a binomial random variable:**

1. Experiment consists of _______________ trials.
2. There are only ______possible outcomes for each trial (S: success or F: failure).
3. The probability of success p remains the ________from trial to trial. ($\underline{q = 1 - p}$)
4. The trials are ______________.
5. The binomial random variable x is the ________________ in n trials.

Example1. A die is tossed ten times. A success is number 2 observed. Let x be the number of times that 2 is observed out of 10 trials. Is x a binomial random variable?
Check the 5 characteristics of a binomial random variable:

Example2. The professor claims that there is an 80% chance that a student in this class will pass a test. Suppose 3 students are randomly selected from this class, define X is the number of students will pass the test out of three students, Is X a binomial random variable?

Example3. Three cards are drawn **without replacement** from a standard deck of 52 cards. A success is getting a diamond. Let x be the number to get the diamond. Is x a binomial random variable?

To find the probability of achieving x successes out of n trials, use binomial probability distribution formula.

Example to find the probability of a binomial random variable:

Example1. The professor claims that there is an 80% chance that a student in this class will pass a test. Suppose 3 students are randomly selected from this class, what is the probability that 2 of these 3 students will pass the test?

The probability that 2 of these 3 students will pass the test is _______ .

Problem The Heart Association claims that only 10% of U.S. adults over 30 years of age meet the President's Physical Fitness Commission's minimum requirements. Suppose four adults are randomly selected and each is given the fitness test.

Use the formula for a binomial random variable to find the probability distribution of x, where x is the number of adults who pass the fitness test. Graph the distribution.

Mean, Variance, and Standard Deviation for a Binomial Random Variable

Mean: $\mu = np$

Variance: $\sigma^2 = npq$

Standard deviation: $\sigma = \sqrt{npq}$

Example1. Let x represent the number of correct guesses on 5 multiple choice questions where each question has 4 answer options and only one is correct.

a. Find the probability distribution for random variable X.

x	0	1	2	3	4	5
P(x)						

b. Find the probability that the # of correct guesses at least 3? (Would it be likely to pass a five-question quiz by blind guessing?)

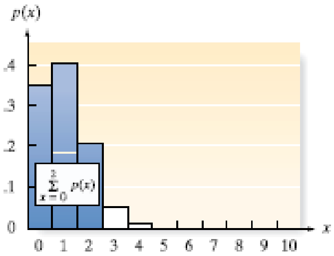
c. Find the mean and standard deviation for the number of correct guesses.

When trials n is large, using formula calculating binomial probability becomes tedious. We can use ______________________________ (Table II, P785-788).

The following is a part of this table.

APPENDIX A Tables 785

TABLE II Binomial Probabilities



Tabulated values are $\sum_{x=0}^{k} p(x)$. *(Computations are rounded at the third decimal place.)*

a. $n = 5$

k \ p	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99
0	.951	.774	.590	.328	.168	.078	.031	.010	.002	.000	.000	.000	.000
1	.999	.977	.919	.737	.528	.337	.188	.087	.031	.007	.000	.000	.000
2	1.000	.999	.991	.942	.837	.683	.500	.317	.163	.058	.009	.001	.000
3	1.000	1.000	1.000	.993	.969	.913	.812	.663	.472	.263	.081	.023	.001
4	1.000	1.000	1.000	1.000	.998	.990	.969	.922	.832	.672	.410	.226	.049

b. $n = 6$

k \ p	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99
0	.941	.735	.531	.262	.118	.047	.016	.004	.001	.000	.000	.000	.000
1	.999	.967	.886	.655	.420	.233	.109	.041	.011	.002	.000	.000	.000
2	1.000	.998	.984	.901	.744	.544	.344	.179	.070	.017	.001	.000	.000
3	1.000	1.000	.999	.983	.930	.821	.656	.456	.256	.099	.016	.002	.000
4	1.000	1.000	1.000	.998	.989	.959	.891	.767	.580	.345	.114	.033	.001
5	1.000	1.000	1.000	1.000	.999	.996	.984	.953	.882	.738	.469	.265	.059

c. $n = 7$

k \ p	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99
0	.932	.698	.478	.210	.082	.028	.008	.002	.000	.000	.000	.000	.000
1	.998	.956	.850	.577	.329	.159	.063	.019	.004	.000	.000	.000	.000
2	1.000	.996	.974	.852	.647	.420	.227	.096	.029	.005	.000	.000	.000
3	1.000	1.000	.997	.967	.874	.710	.500	.290	.126	.033	.003	.000	.000
4	1.000	1.000	1.000	.995	.971	.904	.773	.580	.353	.148	.026	.004	.000
5	1.000	1.000	1.000	1.000	.996	.981	.937	.841	.671	.423	.150	.044	.002
6	1.000	1.000	1.000	1.000	1.000	.998	.992	.972	.918	.790	.522	.302	.068

(continued)

TABLE II Continued

d. $n = 8$

k \ p	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99
0	.923	.663	.430	.168	.058	.017	.004	.001	.000	.000	.000	.000	.000
1	.997	.943	.813	.503	.255	.106	.035	.009	.001	.000	.000	.000	.000
2	1.000	.994	.962	.797	.552	.315	.145	.050	.011	.001	.000	.000	.000
3	1.000	1.000	.995	.944	.806	.594	.363	.174	.058	.010	.000	.000	.000
4	1.000	1.000	1.000	.990	.942	.826	.637	.406	.194	.056	.005	.000	.000
5	1.000	1.000	1.000	.999	.989	.950	.855	.685	.448	.203	.038	.006	.000
6	1.000	1.000	1.000	1.000	.999	.991	.965	.894	.745	.497	.187	.057	.003
7	1.000	1.000	1.000	1.000	1.000	.999	.996	.983	.942	.832	.570	.337	.077

e. $n = 9$

k \ p	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99
0	.914	.630	.387	.134	.040	.010	.002	.000	.000	.000	.000	.000	.000
1	.997	.929	.775	.436	.196	.071	.020	.004	.000	.000	.000	.000	.000
2	1.000	.992	.947	.738	.463	.232	.090	.025	.004	.000	.000	.000	.000
3	1.000	.999	.992	.914	.730	.483	.254	.099	.025	.003	.000	.000	.000
4	1.000	1.000	.999	.980	.901	.733	.500	.267	.099	.020	.001	.000	.000
5	1.000	1.000	1.000	.997	.975	.901	.746	.517	.270	.086	.008	.001	.000
6	1.000	1.000	1.000	1.000	.996	.975	.910	.768	.537	.262	.053	.008	.000
7	1.000	1.000	1.000	1.000	1.000	.996	.980	.929	.804	.564	.225	.071	.003
8	1.000	1.000	1.000	1.000	1.000	1.000	.998	.990	.960	.866	.613	.370	.086

f. $n = 10$

k \ p	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99
0	.904	.599	.349	.107	.028	.006	.001	.000	.000	.000	.000	.000	.000
1	.996	.914	.736	.376	.149	.046	.011	.002	.000	.000	.000	.000	.000
2	1.000	.988	.930	.678	.383	.167	.055	.012	.002	.000	.000	.000	.000
3	1.000	.999	.987	.879	.650	.382	.172	.055	.011	.001	.000	.000	.000
4	1.000	1.000	.998	.967	.850	.633	.377	.166	.047	.006	.000	.000	.000
5	1.000	1.000	1.000	.994	.953	.834	.623	.367	.150	.033	.002	.000	.000
6	1.000	1.000	1.000	.999	.989	.945	.828	.618	.350	.121	.013	.001	.000
7	1.000	1.000	1.000	1.000	.998	.988	.945	.833	.617	.322	.070	.012	.000
8	1.000	1.000	1.000	1.000	1.000	.998	.989	.954	.851	.624	.264	.086	.004
9	1.000	1.000	1.000	1.000	1.000	1.000	.999	.994	.972	.893	.651	.401	.096

(continued)

TABLE II Continued

g. $n = 15$

k \ p	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99
0	.860	.463	.206	.035	.005	.000	.000	.000	.000	.000	.000	.000	.000
1	.990	.829	.549	.167	.035	.005	.000	.000	.000	.000	.000	.000	.000
2	1.000	.964	.816	.398	.127	.027	.004	.000	.000	.000	.000	.000	.000
3	1.000	.995	.944	.648	.297	.091	.018	.002	.000	.000	.000	.000	.000
4	1.000	.999	.987	.838	.515	.217	.059	.009	.001	.000	.000	.000	.000
5	1.000	1.000	.998	.939	.722	.403	.151	.034	.004	.000	.000	.000	.000
6	1.000	1.000	1.000	.982	.869	.610	.304	.095	.015	.001	.000	.000	.000
7	1.000	1.000	1.000	.996	.950	.787	.500	.213	.050	.004	.000	.000	.000
8	1.000	1.000	1.000	.999	.985	.905	.696	.390	.131	.018	.000	.000	.000
9	1.000	1.000	1.000	1.000	.996	.966	.849	.597	.278	.061	.002	.000	.000
10	1.000	1.000	1.000	1.000	.999	.991	.941	.783	.485	.164	.013	.001	.000
11	1.000	1.000	1.000	1.000	1.000	.998	.982	.909	.703	.352	.056	.005	.000
12	1.000	1.000	1.000	1.000	1.000	1.000	.996	.973	.873	.602	.184	.036	.000
13	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.995	.965	.833	.451	.171	.010
14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.995	.965	.794	.537	.140

h. $n = 20$

k \ p	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99
0	.818	.358	.122	.012	.001	.000	.000	.000	.000	.000	.000	.000	.000
1	.983	.736	.392	.069	.008	.001	.000	.000	.000	.000	.000	.000	.000
2	.999	.925	.677	.206	.035	.004	.000	.000	.000	.000	.000	.000	.000
3	1.000	.984	.867	.411	.107	.016	.001	.000	.000	.000	.000	.000	.000
4	1.000	.997	.957	.630	.238	.051	.006	.000	.000	.000	.000	.000	.000
5	1.000	1.000	.989	.804	.416	.126	.021	.002	.000	.000	.000	.000	.000
6	1.000	1.000	.998	.913	.608	.250	.058	.006	.000	.000	.000	.000	.000
7	1.000	1.000	1.000	.968	.772	.416	.132	.021	.001	.000	.000	.000	.000
8	1.000	1.000	1.000	.990	.887	.596	.252	.057	.005	.000	.000	.000	.000
9	1.000	1.000	1.000	.997	.952	.755	.412	.128	.017	.001	.000	.000	.000
10	1.000	1.000	1.000	.999	.983	.872	.588	.245	.048	.003	.000	.000	.000
11	1.000	1.000	1.000	1.000	.995	.943	.748	.404	.113	.010	.000	.000	.000
12	1.000	1.000	1.000	1.000	.999	.979	.868	.584	.228	.032	.000	.000	.000
13	1.000	1.000	1.000	1.000	1.000	.994	.942	.750	.392	.087	.002	.000	.000
14	1.000	1.000	1.000	1.000	1.000	.998	.979	.874	.584	.196	.011	.000	.000
15	1.000	1.000	1.000	1.000	1.000	1.000	.994	.949	.762	.370	.043	.003	.000
16	1.000	1.000	1.000	1.000	1.000	1.000	.999	.984	.893	.589	.133	.016	.000
17	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.996	.965	.794	.323	.075	.001
18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.992	.931	.608	.264	.017
19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.988	.878	.642	.182

(continued)

Note: the entries represent _____________ binomial probabilities,

(Probability that no more than or _________ k successes will occur out of n trials)

Example1, Let x represents the number of correct guesses on 10 multiple choice questions where each question has 5 answer options and only one is correct. Use binomial probability table,

1. find the probability that a person gets at most 2 questions correctly by guessing.

2. find the probability that a person gets at least 6 questions correctly by guessing.

3. find the probability that a person gets 6 questions correctly by guessing

Problem Suppose a poll of 20 voters is taken in a large city. The purpose is to determine x, the number who favor a certain candidate for mayor. Suppose that 60% of all the city's voters favor the candidate.

a. Find the mean and standard deviation of x.

b. Use Table II of Appendix A to find the probability that $x \leq 10$.

c. Use Table II to find the probability that $x > 12$.

d. Use Table II to find the probability that $x = 11$.

e. Graph the probability distribution of x, and locate the interval $\mu \pm 2\sigma$ on the graph.

Example3, The probability that an individual is left-handed is 0.10. In a class there are 15 students.

1. Find the mean and standard deviation of the number of left-handed students in this class.

2. Find the probability that exactly 5 students are left-handed in the class.

3. Find the probability that no more than (at most) 6 students are left-handed?

4. Find the probability that at least two students are left-handed?

4.5 The Poisson Distribution

The _____________**probability distribution** is used to describe the number of rare events that will occur in a specific period of time or in a specific area or volume. (specific unit)

Typical examples of random variables for which the Poisson probability distribution provides a good model are as follows:
1. The number of industrial accidents per month at a manufacturing plant;
2. The number of customer arrivals per unit time at a supermarket checkout counter;
3. The number of death claims received per day by an insurance company;
4. The number of errors per 100 invoices in the accounting records of a company;

Characteristics of a Poisson random variable

1. The experiment consists of ________________________ a certain event occurs during a given unit of time or in a given area or volume or other unit of measurement.
2. The probability that an event occurs in a given unit of time, area, or volume is _________ for all the units.
3. The number of events that occur in one unit of time, area, or volume is _____________ of the number that occur in any other mutually exclusive unit.
4. The ________(or expected) number of events in each unit is denoted by the Greek letter___.

Probability Distribution for a Poisson Random Variable

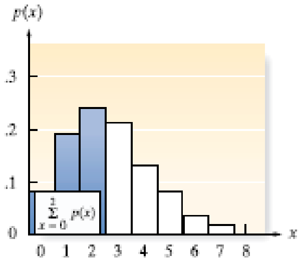
Let x = the number of events that occur in the unit, then the probability that x events will occur during the unit is given by:

____

Note: $e \approx 2.7183$,
λ: _____________ of events during given unit of time, area, volume, etc.

Table III (P789-793), the entries represent _______________ Poisson probabilities

TABLE III Poisson Probabilities



Tabulated values are $\sum_{x=0}^{k} p(x)$. *(Computations are rounded at the third decimal place.)*

λ \ k	0	1	2	3	4	5	6	7	8	9
.02	.980	1.000								
.04	.961	.999	1.000							
.06	.942	.998	1.000							
.08	.923	.997	1.000							
.10	.905	.995	1.000							
.15	.861	.990	.999	1.000						
.20	.819	.982	.999	1.000						
.25	.779	.974	.998	1.000						
.30	.741	.963	.996	1.000						
.35	.705	.951	.994	1.000						
.40	.670	.938	.992	.999	1.000					
.45	.638	.925	.989	.999	1.000					
.50	.607	.910	.986	.998	1.000					
.55	.577	.894	.982	.998	1.000					
.60	.549	.878	.977	.997	1.000					
.65	.522	.861	.972	.996	.999	1.000				
.70	.497	.844	.966	.994	.999	1.000				
.75	.472	.827	.959	.993	.999	1.000				
.80	.449	.809	.953	.991	.999	1.000				
.85	.427	.791	.945	.989	.998	1.000				
.90	.407	.772	.937	.987	.998	1.000				
.95	.387	.754	.929	.981	.997	1.000				
1.00	.368	.736	.920	.981	.996	.999	1.000			
1.1	.333	.699	.900	.974	.995	.999	1.000			
1.2	.301	.663	.879	.966	.992	.998	1.000			
1.3	.273	.627	.857	.957	.989	.998	1.000			
1.4	.247	.592	.833	.946	.986	.997	.999	1.000		
1.5	.223	.558	.809	.934	.981	.996	.999	1.000		

(continued)

Table III	(continued)									
λ \ k	10	11	12	13	14	15	16	17	18	19
7.2	.887	.937	.967	.984	.993	.997	.999	.999	1.000	
7.4	.871	.926	.961	.980	.991	.996	.998	.999	1.000	
7.6	.854	.915	.954	.976	.989	.995	.998	.999	1.000	
7.8	.835	.902	.945	.971	.986	.993	.997	.999	1.000	
8.0	.816	.888	.936	.966	.983	.992	.996	.998	.999	1.000
8.5	.763	.849	.909	.949	.973	.986	.993	.997	.999	.999
9.0	.706	.803	.876	.926	.959	.978	.989	.995	.998	.999
9.5	.645	.752	.836	.898	.940	.967	.982	.991	.996	.998
10.0	.583	.697	.792	.864	.917	.951	.973	.986	.993	.997
	20	21	22							
8.5	1.000									
9.0	1.000									
9.5	.999	1.000								
10.0	.998	.999	1.000							
	0	1	2	3	4	5	6	7	8	9
10.5	.000	.000	.002	.007	.021	.050	.102	.179	.279	.397
11.0	.000	.000	.001	.005	.015	.038	.079	.143	.232	.341
11.5	.000	.000	.001	.003	.011	.028	.060	.114	.191	.289
12.0	.000	.000	.001	.002	.008	.020	.046	.090	.155	.242
12.5	.000	.000	.000	.002	.005	.015	.035	.070	.125	.201
13.0	.000	.000	.000	.001	.004	.011	.026	.054	.100	.166
13.5	.000	.000	.000	.001	.003	.008	.019	.041	.079	.135
14.0	.000	.000	.000	.000	.002	.006	.014	.032	.062	.109
14.5	.000	.000	.000	.000	.001	.004	.010	.024	.048	.088
15.0	.000	.000	.000	.000	.001	.003	.008	.018	.037	.070
	10	11	12	13	14	15	16	17	18	19
10.5	.521	.639	.742	.825	.888	.932	.960	.978	.988	.994
11.0	.460	.579	.689	.781	.854	.907	.944	.968	.982	.991
11.5	.402	.520	.633	.733	.815	.878	.924	.954	.974	.986
12.0	.347	.462	.576	.682	.772	.844	.899	.937	.963	.979
12.5	.297	.406	.519	.628	.725	.806	.869	.916	.948	.969
13.0	.252	.353	.463	.573	.675	.764	.835	.890	.930	.957
13.5	.211	.304	.409	.518	.623	.718	.798	.861	.908	.942
14.0	.176	.260	.358	.464	.570	.669	.756	.827	.883	.923
14.5	.145	.220	.311	.413	.518	.619	.711	.790	.853	.901
15.0	.118	.185	.268	.363	.466	.568	.664	.749	.819	.875
	20	21	22	23	24	25	26	27	28	29
10.5	.997	.999	.999	1.000						
11.0	.995	.998	.999	1.000						
11.5	.992	.996	.998	.999	1.000					
12.0	.988	.994	.987	.999	.999	1.000				
12.5	.983	.991	.995	.998	.999	.999	1.000			
13.0	.975	.986	.992	.996	.998	.999	1.000			
13.5	.965	.980	.989	.994	.997	.998	.999	1.000		
14.0	.952	.971	.983	.991	.995	.997	.999	.999	1.000	
14.5	.936	.960	.976	.986	.992	.996	.998	.999	.999	1.000
15.0	.917	.947	.967	.981	.989	.994	.997	.998	.999	1.000

(Probability that no more than or ________ k events will occur during the unit time)

The Mean, Variance, and Standard Deviation for the Poisson distribution:

Example1: Suppose the number x of a company's employees who are absent on Mondays has a Poisson probability distribution. Assume that the average number of Monday absentees is 2.6.

a. Find the mean and standard deviation of x, the number of employees absent on Monday.

b. Find the probability that fewer than two employees are absent on a given Monday.

c. Find the probability that exactly three employees are absent on a given Monday.

d. Use Table III to find the probability that more than three employees are absent on a given Monday.

Example2. Suppose variable x, the number of cars waiting at a stop sign during 6:00pm—7:00pm has a Poisson probability distribution with average number 15 cars.

a. Find the probability that there are 10 cars waiting at this stop sign at a given 6:00pm-7:00pm period.

b. Find the probability that there are no more than 10 cars waiting at this stop sign at a given 6:00pm-7:00pm period.

c. find the mean and standard deviation of x.

Learning Objective of Chapter 4:

1. Understand random variables: discrete and continuous
2. Describe a probability distribution (possible value of R.V. and corresponding probabilities)
3. Two requirements of probability distribution of a discrete random variable
4. Given a probability distribution of a R.V., Calculate the probabilities, find the mean (expected value) and standard deviation of the discrete random variable
5. Identify Binomial random variable, Calculate the probabilities (using formula and table), find the mean (expected value) and standard deviation of a Binomial random variable
6. Given a Poisson random variable, Calculate the probabilities (using formula and table), find the mean (expected value) and standard deviation of a Poisson random variable