Chapter 6 Sampling Distributions

Parameter and Statistic

A ________________ is a numerical descriptive measure of a **<u>population</u>**. Since it is based on the observations in the population, its value is almost always unknown. (________________)
A __________________ is a numerical descriptive measure of a **<u>sample</u>**. It is calculated from the observations in the sample. (________________)

Note: we will often use the information contained in these __________________________ to make inferences about the _______________________________.

TABLE 6.1 List of Population Parameters and Corresponding Sample Statistics

	Population Parameter	Sample Statistic
Mean:	μ	$\bar{x}$
Variance:	σ^2	s^2
Standard deviation:	σ	s
Binomial proportion:	p	$\hat{p}$

Note that the term *statistic* refers to a *sample* quantity and the term *parameter* refers to a *population* quantity.

6.1 The concept of a Sampling Distributions

The _________________________**of a** ____________________ calculated from a sample of n measurements is **the probability distribution of the statistic.**

*In other words, it is the probability distribution for all of the possible values of the statistic that could result when taking samples of size n.
*Here the _____________ would be a random variable.

Now let consider the sampling distribution of _______________.

Example1: Let random variable X= the number of heads observed when tossing two coins.
The probability distribution for X is:

2. Find the **sampling distribution for the sample mean** $\bar{x}$ when randomly tossing two coins **two times**. (**n = 2**).

3. Find the mean of $\overline{x}$ (mean number of heads observed when randomly tossing two coins two times).

Example2: Let X = the number of boys in a family with three children, then assuming there is equal chance to have a boy and a girl. The probability distribution for the number of boys is:

Find the **sampling distribution for the sample mean** $\overline{x}$ when we look at **two** randomly selected families with three children. (**n = 2**).

Find the mean of $\overline{x}$ (mean number of boys in **two** families with three children).

6.3 The sampling distribution of sample mean and the Central Limit Theorem

Suppose a random sample of n observations has been selected from any population with mean μ and standard deviation σ, the properties of the sampling distribution of sample mean $\overline{x}$:

- Mean of sampling distribution equals mean of sampled population:

- Standard deviation of sampling distribution:

How to find out the sampling distribution of sample mean?

1:

If a random sample of n observations is selected from a population with a _______________ distribution, the sampling distribution of ____________ will be a _____________ distribution.

2. __________________________

If a random sample of n observations is selected from a population (_______________) with mean _____ and standard deviation ______, when the sample size n is sufficiently ________ (__________), the sampling distribution of ____________ will be approximately a _____________ distribution with mean ______________ and standard deviation ______________.

The __________ the sample size, the better will be the ___________ approximation.

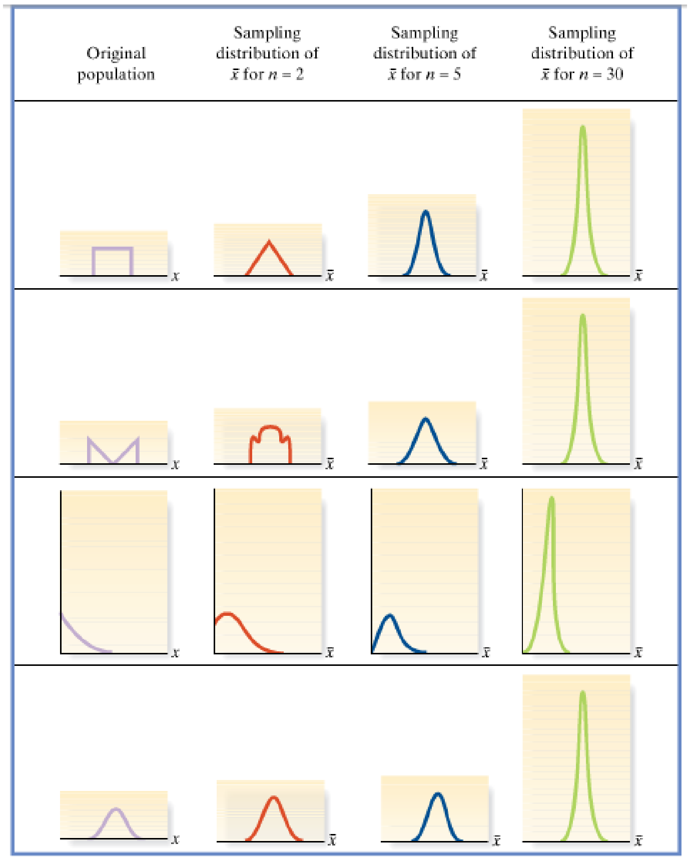

FIGURE 6.10
Sampling distributions of $\bar{x}$ for different populations and different sample sizes

Examples to figure out the sampling distribution of sample mean:

1. The number of violent crimes committed in a day possesses a distribution with a mean of 3.3 crimes per day and a standard deviation of 4 crimes per day. A random sample of 100 days was observed, and the sample mean number of crimes for the sample was calculated. Describe the sampling distribution of the sample mean.

2. Assume that the surface roughness has a normal distribution with mean of 1.8 micrometers and a standard deviation 0.5 micrometer. Determine the sampling distribution of the mean roughness of 20 sampled sections of coated interior pipe.

3. The mean annual income for adult women in one city is $28,520 and standard deviation of the incomes is $5200. The distribution of incomes is skewed to the right. Determine the sampling distribution of the mean for samples of size 121.

A. normally distributed with a mean of $28,520 and a standard deviation of $5200

B. skewed to the right with a mean of $28,520 and a standard deviation of $473

C. normally distributed with a mean of $28,520 and a standard deviation of $260

D. normally distributed with a mean of $28,520 and a standard deviation of $473

From the CLT, we can know the sampling distribution of $\bar{x}$ is normally distributed, then we can work on it as normally distributed random variable.

Steps for finding a Probability corresponding to a Normally distributed sample mean $\bar{x}$:

1. Sketch the _______ curve and indicate the <u>mean</u> μ of random variable (sample mean) $\bar{x}$.
2. Shade the _______ of interest.
3. Convert the boundaries ($\bar{x}$ values) of the shaded area to _______ by using ___________ .
4. Use the ___________ to find the desired area.

Problem Suppose we have selected a random sample of $n = 36$ observations from a population with mean equal to 80 and standard deviation equal to 6. It is known that the population is not extremely skewed.

a. Sketch the relative frequency distributions for the population and for the sampling distribution of the sample mean $\bar{x}$.

b. Find the probability that $\bar{x}$ will be larger than 82.

Example2: A manufacturer of automobile batteries claims that the distribution of the lengths of life of its best battery has a mean of 54 months and a standard deviation of 6 months. Suppose a consumer group decides to check the claim by purchasing a sample of 50 of these batteries.

a. Assuming that the manufacturer's claim is true, describe the sampling distribution of the mean lifetime of a sample of 50 batteries.

b. Assuming that the manufacturer's claim is true, what is the probability the consumer group's sample has a mean of 52 or fewer months?

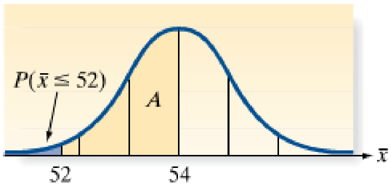

FIGURE 6.13

Sampling distribution of $\bar{x}$ in Example 6.8 for $n = 50$

c. Suppose a consumer group's sample has a mean less than 52 months, do you still think the manufacture's claim is true? Why?

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d.What is the probability that the sample mean has a mean life time more than 55 months?

Example3: Scores on a biology final exam are normally distributed with a mean of 220 and a standard deviation of 24. Determine the percentage of samples of size 9 that will have mean scores within 12 points of the population mean score of 220.

Example4. Testing for Content Accuracy, A brand of water-softener salt comes in packages marked "net weight 40 lb." The company that packages the salt claims that the bags contain an average of 40 lb of salt and the standard deviation is 1.5 lb. Assume that the weight are normally distributed.

a. Determine the probability that the weight of one randomly selected bag of salt will be 39 lb or less.

b. If you bought one bag of salt and it weighed 39 lb or less, would you consider this evidence that the company's claim is incorrect? Why?

c. Determine the probability that the **mean weight** of 10 randomly selected bags of salt will be 39 lb or less.

d. If you bought 10 bags of salt and the mean weight was 39 lb or less, would you consider this evidence that the company's claim is incorrect? Why?

e. Determine the probability that 10 randomly selected bags of salt will have a mean weight within 0.5 lb of the population mean 40 lb.

Learning objective of Chapter 6:

1. Figure out if the sampling distribution of sample mean is with a normal distribution (two theorem: normal population or large sample)
2. Finding the sampling distribution of sample mean is normal, calculate the probabilities. (<,   >, or within)